MARS EXPEDITION OPTIMIZATION

TEAM 9 SEMESTER PROJECT

MAE 598-2015-9

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Abstract

In order to achieve a manned one way mission to Mars, the team proposed a two rocket launch where the payload would then attach in low earth orbit. The first payload would have the food, living space, and water needed to live plus the fuel needed to get from low earth orbit to the Martian surface. The second payload would hold the passengers and the majority of the fuel needed for travel between low earth orbit and the surface of Mars. The team plans to design one rocket model that will carry both of these payloads in separate launches. When using similar stages to the Saturn V rocket, the optimal sizes will allow for a practical launch to Mars as the first colonization.

It is impractical in this scenario to send all of the needed equipment for a set of astronauts in one mission. Resupply missions carrying replacement tools and systems for the mars colony will be launched at most every year. These will use the same rocket design used to get the astronauts to the planet, but will be unmanned during restocking flights.

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1. INTRODUCTION

Ever since humans made it to the moon, there has always been the desire to reach further and explore the rest of the universe. Several satellites, rovers, and probes have been sent out to other stellar bodies, but manned expeditions have not made it past the moon. Most expeditions into space (manned or unmanned) have been launched using only one rocket. Due to the limitations of spaceflight, lifting a large mass into space is very expensive and considerably more difficult.

One major exception to this rule is the international space station, which represents the joint efforts of over 20 module launches (with many more for supplies and crew rotations). By applying the multi-launch and orbital assembly methods used in the ISS, a larger mass could be carried further into space. This has not been attempted due to complexity and budget constraints to the space programs. This paper discusses the research and exploration opportunities provided by utilizing the orbital assembly method to send a manned expedition to Mars. Section 1.1 introduces the general problem statement, and Sections 1.2 through 1.4 introduce the three subsystems that will be optimized.

1.1 General Problem Statement

This analysis considers using two rocket launches for one mission to Mars (unidirectional). One rocket would carry the payload and some fuel into low earth orbit, while the other would carry the booster and remaining fuel needed to travel from earth's orbit to Mars. Once both are in low earth orbit, the two would dock together and make the final trip to the surface of Mars. This analysis optimizes the payload taken to Mars using this method, as well as the launch vehicle to get the two halves into orbit. The two subsystems are linked by dimensional and mass constraints. The subsystems will have tradeoffs of mass and lifting capacity, as well as available storage space and deliverable payload mass. The team hopes to develop an optimally massive rocket to deliver a significantly large payload. This analysis may lead to a local minimum instead of the overall optimal system design.

Once the booster rocket and the mars rocket have both been optimized, a resupply schedule will be made to replace failing systems in the mars colony. There is not enough room on the initial rocket for both the astronauts and all the equipment needed for a self-sustaining civilization. Instead of fizzling out within a year of arrival, supplies will be flown in every year to keep the colony going for 20 years. Launches are expensive and it would be wise to ship several systems at a time and not launch on years that do not need a replacement item sent. This will save money, but is more difficult to predict.

1.2 Orbital Launch Booster (Trevor Slawson, Matthew Catlett)

The function of this subsystem is to carry the two halves of the interplanetary vehicle into low earth orbit and then return to the surface for refueling and reuse. The major tradeoff in this analysis is between the amount of payload mass carried into orbit versus the booster mass required to achieve low earth orbit. NASA and its partners have collected significant data on payload per unit of fuel burned, some of which is available to the public. This system will be optimized for minimal total launch mass as a function of major dimensions. The declaration of an ideal minimum solution does involve significant trades; efforts to reduce size and mass may result in an inability to reach low earth orbit.

1.3 Interplanetary Vehicle (Jenna Lynch)

This subsystem represents the core deliverable package of the mission. It will launch into orbit in two halves, which will rendezvous in low earth orbit and dock together before making the long journey to Mars. The vehicle capsule will serve as an outpost for the astronauts once they land on the Martian surface. The lander will have to contain a sizable mass of equipment, supplies, and living space for the astronauts on the nine month journey, as well as enough fuel to make the trip.

A significant trade-off exists between deliverable payload and fuel requirement; the amount of fuel necessary to make the trip will increase as the deliverable payload mass increases. Additionally, a larger total interplanetary vehicle weight will increase the minimum allowable lifting capacity of the launch boosters, adding complexity and decreasing feasibility of the project. Finally, the distribution of mass between the two halves of the interplanetary vehicle is desired to be fairly equal. This ensures that the OLB lifting capacity is utilized effectively for both launches, minimizing the potential for an excessively massive payload launch followed by a comparatively miniscule one.

1.4 Proactive Supply Launches (Adrian Maranon)

Once a Martian colony is established it will inevitably be faced with equipment failures. The proactive supply launch subsystem is a plan that will ensure that replacement gear is delivered in an optimal manner. A 22 year launch period was selected to model the Mars One colonization plan. Mars One is a private spaceflight company that intends to land the first humans on mars. The objective of the proactive supply launch subsystem is to minimize the number of resupply launches needed for the settlement to reach self-sustainability. A total of 21 annual resupply launches are available. The duration of each trip from Earth to Mars is simplified to be one year. All of the landers for this subsystem will be unmanned, and will be used solely for the purpose of resupplying the Mars base.

Establishing a permanent Mars colony will rely heavily on in-situ resource utilization (ISRU). According to NASA in-situ resource utilization will enable the affordable establishment of extraterrestrial exploration and operations by minimizing the materials carried from Earth. The cargo is broken down into 7 critical assemblies: the oxygen generation assembly, the carbon dioxide removal assembly, the common cabin air assembly, the urine processor assembly, the water processor assembly, the carbon dioxide reduction assembly, and ISRU assembly. Section 5.1.1 discusses these systems, their function, their respective mass (kg), their respective volume (m^3), and the mean time between failures (MTBF) for each individual component. It is assumed that the MTBF values represent the working lifespan of each component.

2. NOMENCLATURE

Since the scope of this project spans several subsystems (each of which encompass multiple dimensions and components), it is necessary to define the general nomenclature for the project. The general nomenclature is shown in Table 2-1, with the subsystem-specific nomenclature shown in Tables 2-2 through 2-4. Note that general nomenclature abbreviations in Table 2-1 are found frequently in the body of the report, and care should be taken accordingly to possess a familiarity with them before proceeding.

Table 2-1: General Nomenclature

Full Name	Abbreviation
Orbital Launch Booster	OLB
Interplanetary Vehicle	IPV
IPV Payload Half	IPV1
IPV Booster Half	IPV2
Proactive Supply Launches	PSL
Trans-Martian Insertion	TMI
Low Earth Orbit	LEO
Low Mars Orbit	LMO
Martian Surface	MS
Apollo Command Module	ACM

Table 2-2: OLB Nomenclature

Variable Name	Symbol	Units	Variable Name	Symbol	Units
Mass of the OLB	m_{OLB}	kg	Total Fuel Volume	$V_{F,i}$	m³
Mass of Thrusters	$m_{T\ total,i}$	kg	Structural Surface Area	$A_{S,i}$	m²
Mass of Fuel	$m_{F,i}$	kg	Ratio of Structure	RS_i	kg/m²
Mass of Structure	$m_{S,i}$	kg	Thruster Mass	$m_{T,i}$	kg
Height	h_i	m	Number of Thrusters	$n_{T,i}$	-
Radius	r_i	m	Change in Velocity	Δv_i	m/s
Fuel Ratio WRT Mass	MFR_i	kg/kg	Gross Vehicle Weight	GVW_i	kg
Fuel Ratio WRT Volume	VFR_i	m³/m³	Empty Vehicle Weight	EVW_i	kg
Density of Oxidizer	ρ_{Ox}	kg/m³	Thrust Transient	$T(t)$	kN
Density of Fuel	ρ_{Fuel}	kg/m³	Mass Transient	$m(t)$	kg
Average Fuel Density	$\rho_{F,i,avg}$	kg/m³	Maneuver Time	t_i	s
Stage Number	i	-			

Table 2-3: IPV Nomenclature

Variable Name	Symbol	Units	Variable Name	Symbol	Units
Mass of the Payload	$m_{Payload}$	kg	IPV Structural Mass	m_{IPVS}	kg
Mass of *n*	m_n	kg	ACM Structural Mass	m_{ACMS}	kg
Height of *n*	h_n	m	Rocket Mass-Area Ratio	RS	kg/m^2
Density of *n*	ρ_n	kg/m^3	Force of Thruster	T	kN
Radius of IPV	r	m	Mass Flow Rate of Engine	$\dot{m}_{Engine}$	kg/s
Number of Astronauts	n_{astro}	-	Change in Velocity	Δv_i	m/s
Average Human Mass	m_{person}	kg	Burn Phase	i=[1:3]	-
Fuel Remainder Ratio	FRR	-			

Table 2-4: PSL Nomenclature

Variable Name	Symbol	Units
Mass of Oxygen Generation Assembly	m_{OG}	*kg*
Mass of Carbon Dioxide Removal Assembly	m_{CDR}	*kg*
Mass of Common Cabin Air Assembly	m_{CCA}	*kg*
Mass of Urine Processor Assembly	m_{UP}	*kg*
Mass of Water Processor Assembly	m_{WP}	*kg*
Mass of CO_2 Reduction Assembly	m_{CO2}	*kg*
Mass of ISRU	m_{ISRU}	*kg*
Volume of Oxygen Generation Assembly	V_{OG}	m^3
Volume of Carbon Dioxide Removal Assembly	V_{CDR}	m^3
Volume of Common Cabin Air Assembly	V_{CCA}	m^3
Volume of Urine Processor Assembly	V_{UP}	m^3
Volume of Water Processor Assembly	V_{WP}	m^3
Volume of CO_2 Reduction Assembly	V_{CO2}	m^3
Volume of ISRU	V_{ISRU}	m^3
MTBF of Oxygen Generation Assembly	$MTBF_{OG}$	*years*
MTBF of Carbon Dioxide Removal Assembly	$MTBF_{CDR}$	*years*
MTBF of Common Cabin Air Assembly	$MTBF_{CCA}$	*years*
MTBF of Urine Processor Assembly	$MTBF_{UP}$	*years*
MTBF of Water Processor Assembly	$MTBF_{WP}$	*years*
MTBF of CO_2 Reduction Assembly	$MTBF_{CO2}$	*years*
MTBF of ISRU	$MTBF_{ISRU}$	*years*
Maximum Available Mass of IPV	M_{IPV}	*kg*
Maximum Available Volume of IPV	V_{IPV}	m^3

Index Meaning	Value
Assembly/ Component # *(n)*	*N=7*
Launch # *(i)*	*I = 21*
Timeframe *(t)*	*T = 21*
IxN Matrix	*X*

3. ORBITAL LAUNCH BOOSTER (OLB)

3.1 Mathematical Model

The objective function of this subsystem is to minimize the overall mass of the OLB. The mass of the OLB is determined by the radius and height of the stages, as well as the pressure of the liquid fuel required to achieve LEO. This equation is a simple summation of the masses of each stage's various components. Each stage will have *n* thrusters, fuel, and a surrounding structure (2.1).

$$m_{OLB} = \sum_{i=1}^{N} m_{T\ total,i} + m_{F,i} + m_{S,i} \tag{2.1}$$

The mass of the fuel is associated with the volume of fuel available based on the overall size of each cylindrical stage section. Each stage requires a certain quantity of fuel and oxidizer to be mixed according to a certain weight ratio (2.2).

$$MFR_1 = 2.27,\ \ MFR_2 = 5.50 \tag{2.2}$$

The first stage relies on the RP-1 propellant, a form of refined kerosene with a density between 810 and 1015 kg/m³ (a mean value of 915 kg/m³ is assumed for this model). The second stage uses liquid hydrogen (LH_2), which has a density of 71 kg/m³. The oxidizer in both cases is liquid oxygen (LOX) with a density of 1141 kg/m³.

For the purposes of this model, the mass of fuel will be approximated based on volume available in each stage structure. It is necessary to define the volumetric mixing ratios in order to determine how much fuel each stage can store (2.3).

$$VFR_1 = 1.82,\ \ VFR_2 = 0.34 \tag{2.3}$$

These volumetric ratios shed light on an interesting design factor with respect to the second stage: while the oxidizer mass will be considerably greater than the fuel mass, the fuel will occupy nearly three times the volume. Since the fuel mass will be approximated by volume, a representative weighted mean density must be established using the volumetric ratio (2.4).

$$\rho_{F,i,avg} = \frac{1}{VFR_i + 1}\left(VFR_i \cdot \rho_{Ox} + \rho_{Fuel}\right) \tag{2.4}$$

This average fuel density was initially intended to be applied directly to the cylindrical volume of the stage. However, these results were inconsistent with the fuel mass of the Saturn V stages. Further research indicates that both stages utilized cylindrical tanks with spherical ends. This tank geometry is approximated by adding the volume of two spheres and one cylinder (2.5).

$$V_{F,i} = \pi\left(h_i r_i^2 + \frac{8}{3} r_i^3\right) \tag{2.5}$$

The total fuel mass of a stage (2.6) is found by combining equations 2.4 and 2.5.

$$m_{F,i} = \frac{\pi}{VFR_i + 1}\left(VFR_i \cdot \rho_{Oxidizer} + \rho_{Fuel}\right)\left(h_1 r_i^2 + \frac{8}{3} r_i^3\right) \tag{2.6}$$

The structural mass is complex and highly dependent on internal supports and equipment that is beyond the scope of this project to itemize and model. Instead, a general relationship of mass against surface area for each stage of the Saturn V is used. The surface area of a cylinder is used as an approximation for the stage geometry (2.7).

$$A_{S,i} = 2\pi r_i(h_i + r_i) \tag{2.7}$$

Since the height, radius, and empty vehicle mass (EVM) of the Saturn V first and second stage boosters is readily available the desired relationship between surface area and mass, called the Ratio of Structure (2.8) is found by dividing the EVM by equation 2.7 and substituting in the respective height and radius.

$$RS_i = EVM_{Saturn\,V,i}\left[2\pi r_{Saturn\,V,i}\left(h_{Saturn\,V,i} + r_{Saturn\,V,i}\right)\right]^{-1} \tag{2.8}$$

The mass of an OLB stage can then be found by multiplying the respective RS value by the surface area of that OLB stage (2.9)

$$m_{S,i} = RS_i[2\pi r_i(h_i + r_i)] \tag{2.9}$$

The mass of the thrusters for each stage is based on existing rocket motor data. Thruster selection matches the Saturn V, and therefore all individual masses are known and must simply be multiplied by the number of thrusters per stage (2.10).

$$m_{T\,total,i} = m_{T,i} \cdot n_{T,i} \tag{2.10}$$

3.1.1 Constraints

The constraints for the OLB comprehensively constrain its dimensions and also enforce the lower bound of fuel required to accomplish its mission of delivering the two halves of the IPV payload to LEO. The size constraints of the OLB bound the design above by the rough Saturn V dimensions (2.11 through 2.14). Additionally, the radial constraint (2.15) ensures that the rocket will either be a constant diameter or will taper down in each subsequent stage. All the dimensions are bounded below by a minimum requirement of zero (2.16) to ensure that the model stays within feasible range.

$$h_1 \leq 42 \text{ m} \tag{2.11}$$

$$h_2 \leq 25 \text{ m} \tag{2.12}$$

$$h_3 \leq 15.8 \text{ m} \tag{2.13}$$

$$r_1 \leq 5.05 \text{ m} \tag{2.14}$$

$$r_3 \leq r_2 \leq r_1 \tag{2.15}$$

$$h_1, h_2, h_3, r_1, r_2, r_3 \geq 0 \text{ m} \tag{2.16}$$

These constraints reflect both the production cost and structural viability of the mission. Since structural and financial factors are not included in the model, these constraints ensure that the OLB exists within the bounds of previous successful NASA operations.
The remaining constraints are based on the amount of energy required to place the IPV halves into LEO. The amount of energy required to change position via rocket propulsion is proportional to the change in velocity of a rocket, or literally the "delta-v" (2.17).

$$\Delta v = \int_{t_0}^{t_1} \frac{|T(t)|}{m(t)} dt \tag{2.17}$$

The v value represents a change in velocity via the firing of a rocket (maneuver). The thrust term for each stage (2.18) is simply the product of the thrust per engine and the number of engines. Since the mass flow rate is assumed to be constant, the mass transient (2.19) is a downwards sloping line between the gross vehicle weight (GVW) at time zero and the GVW minus fuel usage at time t_i when the maneuver is complete.

$$|T(t)| = n_{T,i} \cdot T_i \tag{2.18}$$

$$m(t) = GVW_i - \frac{m_{F,i}}{t_i} t \tag{2.19}$$

The delta-v for a stage can be found by substituting 2.18 and 2.19 into equation 2.17 and performing the integration. The result is a time dependent delta-v equation for each stage (2.20).

$$\Delta v_i = \frac{n_{T,i} \cdot T_i \cdot t_i}{m_{F,i}} \ln\left(\frac{m_{E,i} + m_{F,i}}{m_{F,i}}\right) \tag{2.20}$$

The empty mass $m_{E,i}$ for each stage represents the remaining mass after the burn is completed. This value changes for each stage. For example, the empty mass of the vehicle will consist of the entire mission weight minus the first stage fuel at the end of the first burn. However, the empty mass for the second burn will include only the mass of the second stage and payload minus the second stage fuel. The next constraint for this subsystem is based on the known energy requirement to reach orbit; the total delta-v of the OLB must be a minimum of 9 km/s in order to achieve LEO (2.21).

$$\Delta v_i = \sum_{i=1}^{3} \frac{n_{T,i} \cdot T_i \cdot t_i}{m_{F,i}} \ln\left(\frac{m_{E,i} + m_{F,i}}{m_{F,i}}\right) \geq 9 \text{ km/s} \tag{2.21}$$

Optimization of a rocket is governed by the principles of fuel reduction. That is, the difference in mass between the beginning and end of a maneuver should ideally be as small as possible while still achieving the desired result. One clever approach to mass reduction during burns is the utilization of staging, which allows for the jettison of mass after a burn is executed. This dead-weight removal allows for a significant improvement in the overall Δv of the OLB, which is the main reason two stages are used. Indeed, due to the tyranny of the rocket equation it would be practically impossible to achieve LEO with a single-stage rocket.

The OLB will be constrained to a payload mass that is equivalent to the most massive IPV section (2.22). This constraint originates directly from Section 4 and is represented as a component of the empty mass in each OLB delta-v calculation. Since the maximum lift capacity of the OLB is loosely tied to the Saturn V specifications, this also serves as a limiting factor to the maximum allowable weight of the IPV.

$$m_{payload} \geq \max(m_{IPV1}, m_{IPV2}) \tag{2.22}$$

In order to achieve vertical flight, each stage will have to generate a positive acceleration at the start of its respective burn. This can be modeled using Newton's second law (2.23). The stage number (*N*) represents which stage will be performing the burn. The summation of masses represents the total mass that stage *N* will have to lift.

$$\sum F = n_{T,N} \cdot T_N - g\left(m_{payload} + \sum_{i=N}^{3} m_{T\ total,i} + m_{F,i} + m_{S,i}\right) \geq 0 \tag{2.23}$$

Another way to describe the acceleration constraint is via thrust to weight ratio, which is obtained simply by rearranging Equation 2.23.

$$\frac{n_{T,N} \cdot T_N}{g\left(m_{payload} + \sum_{i=N}^{3} m_{T\ total,i} + m_{F,i} + m_{S,i}\right)} \geq 1 \tag{2.24}$$

The effect of acceleration on the astronauts is also a primary concern. Since the thrust is assumed to be linear, the maximum acceleration for any given burn will occur at the end, just as the fuel runs out. This can also be described via force equilibrium (2.25). Rearranging the terms puts the constraint in terms of acceleration (2.26), with the maximum allowable sustained acceleration for the human body being 6 g's.

$$\sum F = n_{T,N} \cdot T_N - g \cdot m_{E,N} = a_N \cdot m_{E,N} \tag{2.25}$$

$$\frac{n_{T,N} \cdot T_N - g \cdot m_{E,N}}{m_{E,N}} = a_N \leq 6g \tag{2.26}$$

3.1.2 Design Variables and Parameters

As explained in the previous sections, all of the quantities related to the objective function and constraints can be derived from dimensional quantities or variables that are treated as inputs from other subsystems. Accordingly, the design variables for the OLB are as follows:

$h_1 \equiv$ height of stage 1
$h_2 \equiv$ height of stage 2
$h_3 \equiv$ height of stage 3
$r_1 \equiv$ radius of stage 1
$r_2 \equiv$ radius of stage 2
$r_3 \equiv$ radius of stage 3

Many of the reference values used for calculations are derived from the Saturn V stages, which is a proven launch that reached LEO with a payload of significant mass. Additionally, as stated in Section 3.1.1 the Saturn V dimensions were used as upper bounds on the design variables. Therefore, it is fitting and appropriate to use the same values to verify the other assumptions made while constructing the model. The results of this validation are shown in Table 3.1.2-1 below. Saturn V values are shown in Table 3.1.2-2 for comparison. Note that two launches of the OLB are capable of lifting a total of nearly 236,000 kg to LEO, which is more than half the mass of the International Space Station.

Note that the fuel masses are nearly identical to the actual values. This lends strong support to the model. In addition, the total Delta-v is roughly 8.4 km/s, which is expected since the Saturn V had an additional third stage that was not utilized in the OLB model during this phase of development. It was decided to implement the third stage in order to accommodate the extra 0.6 km/s needed.

Table 3.1.2-1: OLB Verification Values

Objective Function		
mtot =	2872627	kg
Intermediates		
mf_1 =	2271781	kg
ms_1 =	88235	kg
mnT_1 =	41765	kg
mf_2 =	431210	kg
ms_2 =	30691	kg
mnT_2 =	8945	kg

Design Variables		
h_1 =	33.46	m
r_1 =	5.05	m
h_2 =	21.4	m
r_2 =	5.05	m

Addtl. Constraint Variables		
Δv =	8370.635	m/s
Intermediates		
Δv_1 =	3704.755	m/s
Δv_2 =	4665.879	m/s
mE_1 =	718845.9	kg
mE_2 =	157635.9	kg

Imports		
mT_1 =	8353	kg
mT_2 =	1789	kg
n_1 =	5	
n_2 =	5	
T_1 =	34020	
T_2 =	4400	
t_1 =	174	
t_2 =	347	
mP =	118000	kg
ρLOX =	1141	kg/m^3
ρLH2 =	70.85	kg/m^3
ρRP1 =	915	kg/m^3
MRF_1 =	2.27	kg/kg
MRF_2 =	5.50	kg/kg
VRF_1 =	1.82	m^3/m^3
VRF_2 =	0.34	m^3/m^3
RS_1 =	72.21	kg/m^2
RS_2 =	36.57	kg/m^2

Table 3.1.2-2: Saturn V Comparison Values

Saturn V Constants					
Stage 1			Stage 2		
L =	42.1	m	L =	24.8	m
LST =	33.46	m	LST =	21.8	m
R =	5.05	m	R =	5.05	m
GVW =	2290000	kg	GVW =	496200	kg
EVW =	130000	kg	EVW =	40100	kg
STW =	88235	kg	STW =	31155	kg
NFW =	2160000	kg	NFW =	456100	kg
tB =	165	s	tB =	360	s
nT =	5	thrusters	nT =	5	thrusters
Rocketdyne F-1			Rocketdyne J-2		
L =	8.64	m	L =	3	m
M =	8353	kg	M =	1789	kg
mdot =	2618.182	kg/s	mdot =	248.5559	kg/s
T =	6804	kN	T =	880	kN

3.1.3 Summary Model

Minimize:

$$m_{OLB} = \sum_{i=1}^{N} m_{T,i} \cdot n_{T,i} + \frac{\pi}{VFR_i + 1}(VFR_i \cdot \rho_{Ox} + \rho_{Fuel})\left(h_1 r_i^2 + \frac{8}{3} r_i^3\right) + RS_i[2\pi r_i(h_i + r_i)]$$

Subject To:

$g_1 = -h_1 \leq 0\ m$ $\quad g_2 = h_1 \leq 42\ m$ $\quad g_3 = -r_1 \leq 0\ m$ $\quad g_4 = r_1 \leq 5.05\ m$

$g_4 = -h_2 \leq 0\ m$ $\quad g_5 = h_2 \leq 25\ m$ $\quad g_7 = -r_2 \leq 0\ m$ $\quad g_8 = r_2 \leq r_1$

$g_7 = -h_3 \leq 0\ m$ $\quad g_9 = h_3 \leq 15.8\ m$ $\quad g_{11} = -r_3 \leq 0\ m$ $\quad g_{12} = r_3 \leq r_2$

$$\mathrm{g}_{13} = -\sum_{i=1}^{3} \frac{n_{T,i} \cdot T_i \cdot t_i}{m_{F,i}} \ln\left(\frac{m_{E,i} + m_{F,i}}{m_{F,i}}\right) \leq -9\ \mathrm{km/s}$$

$$g_{13+N} = \frac{n_{T,N} \cdot T_N}{g(m_{payload} + \sum_{i=N}^{3} m_{T\ total,i} + m_{F,i} + m_{S,i})} \geq 1 \qquad N=[1:3]$$

$$g_{16+N}\ \frac{n_{T,N} \cdot T_N - g \cdot m_{E,N}}{m_{E,N}} = a_N \leq 6g \qquad N=[1:3]$$

Design Variables:

$h_1 \equiv$ height of stage 1
$h_2 \equiv$ height of stage 2
$h_3 \equiv$ height of stage 3
$r_1 \equiv$ radius of stage 1
$r_2 \equiv$ radius of stage 2
$r_3 \equiv$ radius of stage 3

3.2 Model Analysis

The OLB subsystem model outlined in Section 3.1 was subjected to a monotonicity analysis in order to check for active constraints and well-boundedness. The results of this analysis are shown in Table 2.4-1.

This analysis shows that the system is well bounded both above and below. However, it is not apparent that any specific constraints will be active. Some insight is gained regarding the grouping of constraints. For example, either g1 or g9 will be active because one of the two must be active in order to constrain the objective function with respect to the height of the first stage. One particular point of interest is the relationship between r_1, r_2, and r_3, which allows for the three to be equivalent. In this case all radial constraints could be active, which would change the overall mass required for delta-v.

Unfortunately, due to the integer constraints on number of thrusters, it is difficult to predict whether or not a given constraint will be nearly active or actually active.

Table 2.4-1: Monotonicity Analysis

	h_1	h_2	h_3	r_1	r_2	r_3
f_1	+	+	+	+	+	+
g_1	-					
g_2	+					
g_3				-		
g_4				+		
g_5		-				
g_6		+				
g_7					-	
g_8					+	
g_9			-			
g_{10}			+			
g_{11}						-
g_{12}						+
g_{13}	-	-	-	-	-	-
g_{14}	-	-	-	-	-	-
g_{15}		-	-		-	-
g_{16}			-			-
g_{17}	-	-	-	-	-	-
g_{18}		-	-		-	-
g_{19}			-			-

3.3 Optimization Study

The final model was coded and solved via the GRG Nonlinear Excel solver. Input values in the form of a 118,000 kg payload and using five thrusters for each stage were used. This case was considered in an attempt to locate possible local minima. The results of this run are shown in Figure 3.3-1. The answer report is shown in Figure 3.3-2.

The optimal solution is notably bounded by the feasible region constraints on the variables; every variable is maximized with the exception of the stage 1 height. Additionally, the entire solution is bounded below by the delta-v constraint of 9 km/s to reach LEO. Accordingly, this is a boundary solution. It is not expected to be a global solution due to the strong initial-guess dependence discussed in Section 3.4.

The numerical results of this case study are more positive than the last. The OLB is able to match the lifting capacity of the Saturn V. The constraints from case 1 remain bound at the same locations, and the radius and height of the second stages are also locked at the upper bound. The height of the first stage is still not maximized indicating that the relative efficiency is still preserved.

Objective Function		
$mtot$ =	2923316	kg
Intermediates		
mf_1 =	2053805	kg
ms_1 =	82358.989	kg
mnT_1 =	41765	kg
mf_2 =	530223	kg
ms_2 =	34868	kg
mnT_2 =	8945	kg
mf_2 =	141651	kg
ms_2 =	27911	kg
mnT_2 =	1789	kg
TFP =	93.2	%
Height =	86.3	m
Dia. =	10.1	m
EVM =	197637	kg
GVM =	2923316	kg

Design Variables		
h_1 =	30.89543	m
r_1 =	5.05	m
h_2 =	25	m
r_2 =	5.05	m
h_2 =	15.8	m
r_2 =	3.3	m

Addtl. Constraint Variables		
Δv =	9000	m/s
Intermediates		
Δv_1 =	2923.224	m/s
Δv_2 =	3371.314	m/s
Δv_2 =	2705.461	m/s
mE_1 =	987511.1	kg
mE_2 =	333164	kg
mE_2 =	147700	kg

Imports		
mT_1 =	8353	kg
mT_2 =	1789	kg
mT_2 =	1789	
n_1 =	5	
n_2 =	5	
n_2 =	1	
T_1 =	34020	
T_2 =	4400	
T_2 =	1000	
t_1 =	157	
t_2 =	427	
t_2 =	570	
mP =	118000	kg
ρLOX =	1141	kg/m^3
$\rho LH2$ =	70.85	kg/m^3
$\rho RP1$ =	915	kg/m^3
MRF_1 =	2.27	kg/kg
MRF_2 =	5.50	kg/kg
VRF_1 =	1.82	m^3/m^3
VRF_2 =	0.34	m^3/m^3
RS_1 =	72.21	kg/m^2
RS_2 =	36.57	kg/m^2
RS_2 =	70.48	kg/m^2

Figure 3.3-1: OLB Results

Saturn V Constants								
Stage 1			Stage 2			Stage 3		
L =	42.1	*m*	L =	24.8	*m*	L =	18.8	*m*
LST =	33.46	*m*	LST =	21.8	*m*	LST =	15.8	*m*
R =	5.05	*m*	R =	5.05	*m*	R =	3.3	*m*
GVW =	2290000	*kg*	GVW =	496200	*kg*	GVW =	123000	*kg*
EVW =	130000	*kg*	EVW =	40100	*kg*	EVW =	29700	*kg*
STW =	88235	*kg*	STW =	31155	*kg*	STW =	27911	*kg*
NFW =	2160000	*kg*	NFW =	456100	*kg*	NFW =	93300	*kg*
tB =	165	*s*	tB =	360	*s*	tB =	500	*s*
nT =	5	*thrusters*	nT =	5	*thrusters*	nT =	1	*thrusters*
Rocketdyne F-1			Rocketdyne J-2			Rocketdyne J-2		
L =	8.64	*m*	L =	3	*m*	L =	3	*m*
M =	8353	*kg*	M =	1789	*kg*	M =	1789	*kg*
mdot =	2618.182	*kg/s*	mdot =	248.5559	*kg/s*	mdot =	248.5559	*kg/s*
T =	6804	*kN*	T =	880	*kN*	T =	1000	*kN*

Figure 3.3-1: Case 2 Saturn V Comparison Information

3.4 Parametric Study

The next point of interest is to determine the maximum possible payload that can be delivered to LEO, as this will impact the total mass of the IPV and ultimately the deliverable payload to the surface of Mars. The variation of payload mass allowed for a parametric study to be performed. The results are shown in Figures 3.4-1a and 3.4-1b. Based on numerical experiments using the GLG nonlinear excel solver, the resulting theoretical maximum LEO payload for the OLB is approximately 109 metric tons.

At this point a notable issue is observed: the nonlinearity introduced by the integer constraints presents considerable challenges for the excel solver. varying initial dimension guesses confirm this suspicion, as numerous local optimal solutions are found depending on the guess. In some cases the solver will fail entirely. Other strange nonlinearity effects are observed with the dimensions, which show that the OLB minimum diameter for the upper stages shrinks to almost nothing for a payload of roughly 30-100 metric tons.

The significant nonlinearity and strong dependence on initial guesses of the OLB introduces several issues for system integration and determination of a global optimum. Allowing the solver to automatically vary the number of thrusters per stage results in a much more complex problem, which drastically increases the computational challenges. Ultimately, this study indicates the need for alternative solution methods to be utilized.

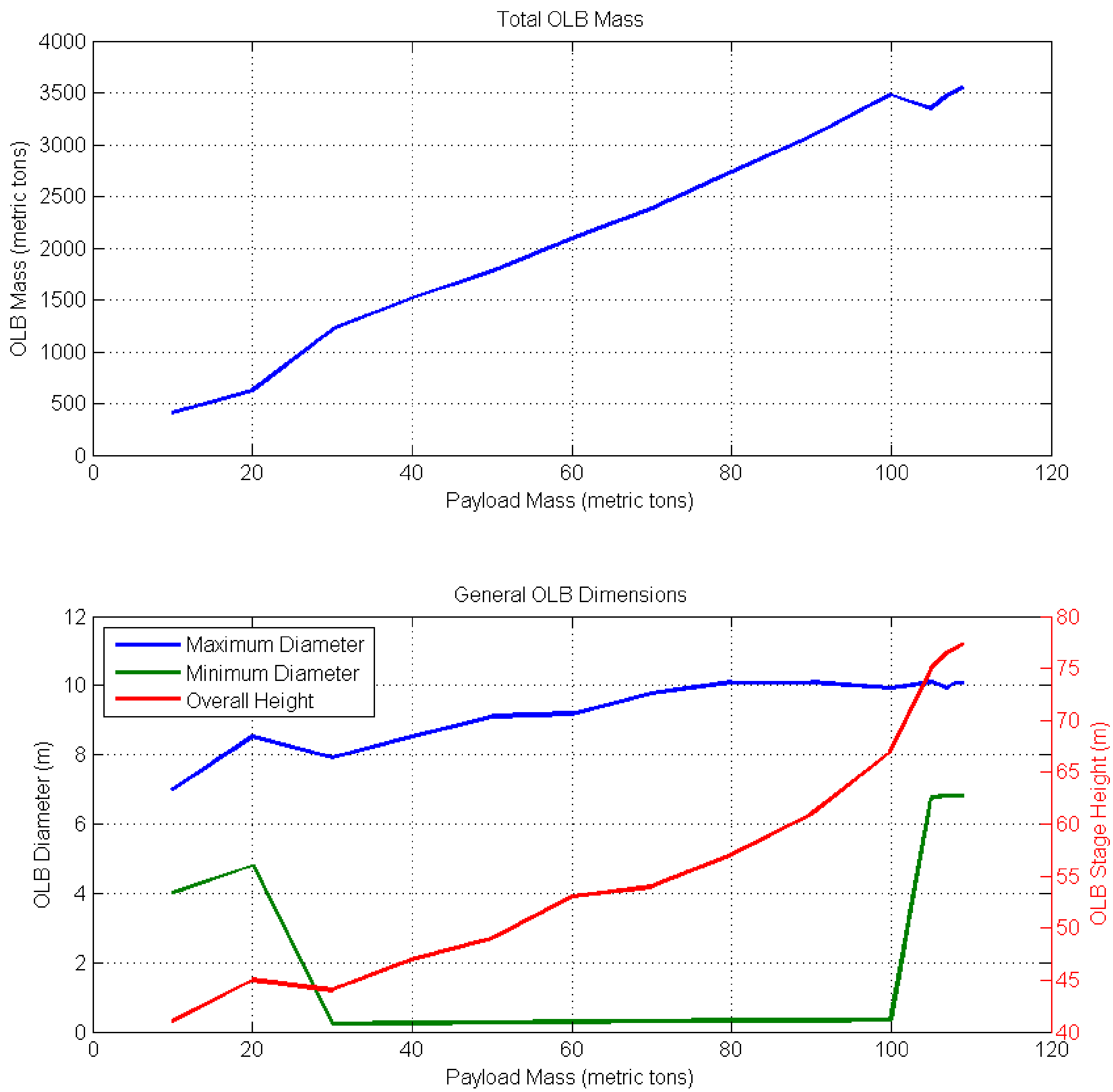
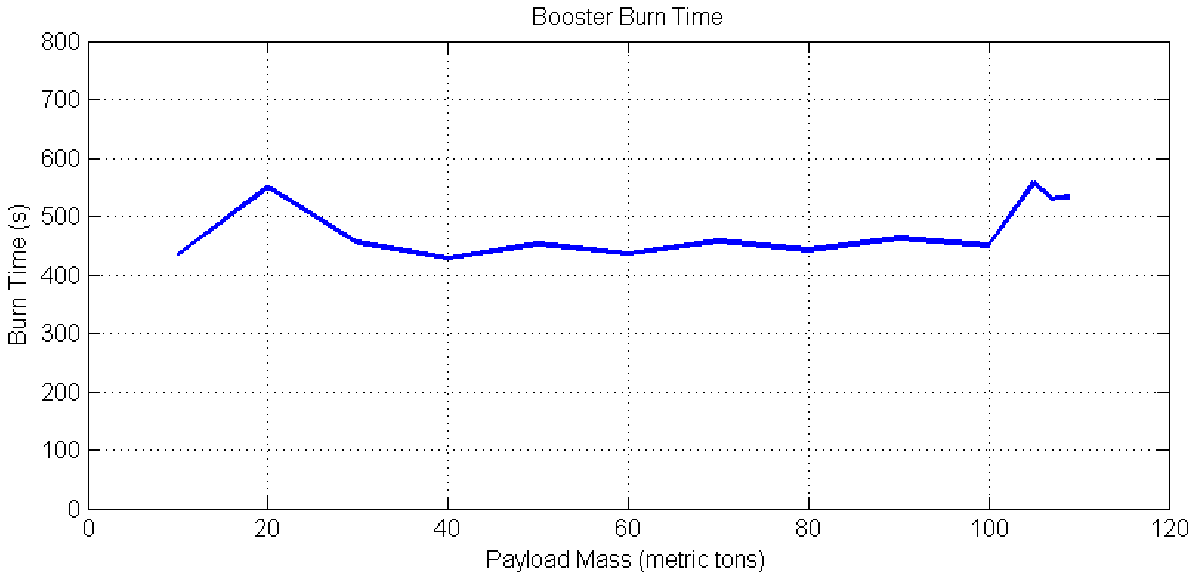


Figure 3.4-1a: OLB Parametric Study (Booster Mass and Major Dimensions)



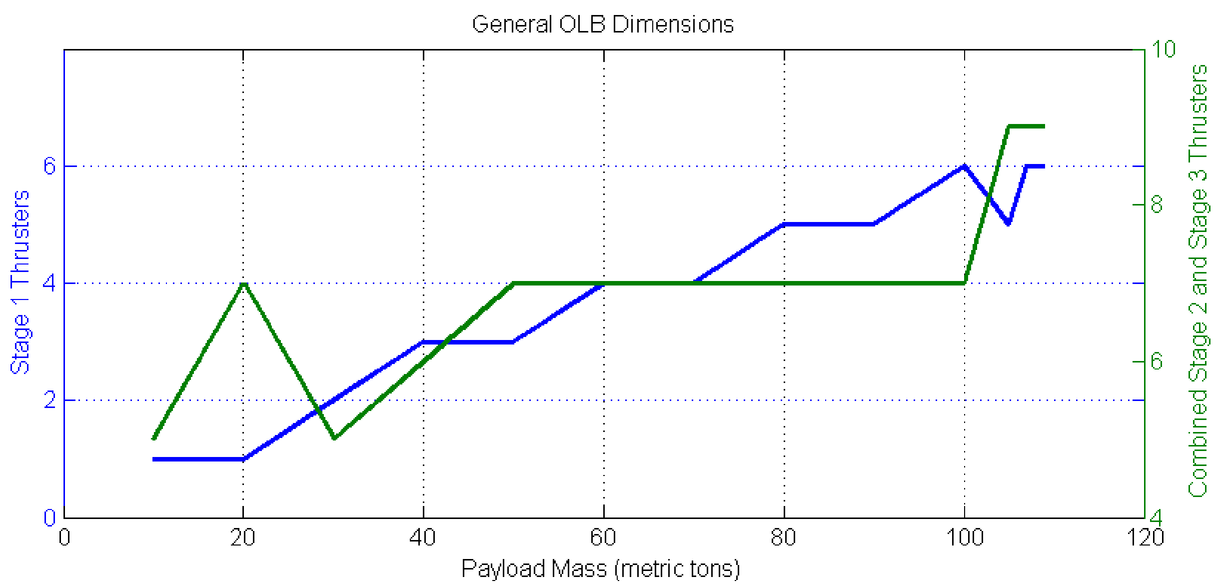


Figure 3.4-1b: OLB Parametric Study (Burn Time and No. of Thrusters)

3.5 Discussion of Results

The main objective of the OLB subsystem optimization is to validate the final model prior to its implementation with the other systems. This validation arises from a detailed comparison with the Saturn V specifications. The results are shown in Table 3.5-1.

Table 3.5-1: OLB Case 2 v. Saturn V Comparison

Value	Saturn V	OLB	Net % Change
Height of All Stages	85.7 m	86.3 m	+ 0.7%
Maximum Diameter	10.1 m	10.1 m	0.0%
EVM	183600 kg	197637 kg	+ 7.6%
GVM	2909200 kg	2923316 kg	+ 0.5%
Percent Fuel by Mass	94.1%	93.2%	- 0.9%
Payload to LEO	118,000 kg	118,000 kg	0.0%

These results are overwhelmingly positive. The EVM is only slightly larger than the target value, with overall height and GVM both within 1.0% of the Saturn V specifications. The percent fuel by mass is well within the feasible bounds for space travel. Considering the omission of structural design considerations and aerodynamic forces this model is sufficient for the purposes of this optimization analysis. Unfortunately, due to the nonlinear effects and integer constraints it is difficult to determine exact design rules. The results do suggest that the more fuel efficient upper stages should be utilized as much as possible, with the relatively inefficient first stage compensating as needed.

One talking point is the percent fuel by mass discrepancy. While a difference of 0.9% seems trivial, this is not the case when rocketry is involved. This fact is illustrated in Don Pettit's discussion of the tyranny of the rocket equation, wherein he indicates the typical percent fuel by mass ratios for a rocket is typically 94% for a kerosene-oxygen rocket like the Saturn V or OLB. From the perspective of a rocket engineer this error is more significant, but still within a sensible range.

4. INTERPLANETARY VEHICLE (IPV)

4.1 Mathematical Model

The objective function of this subsystem is to maximize the overall deliverable payload mass of the IPV. The mass of the final payload is determined by the volume of its various compartments, and the mass of supplies, equipment, personnel, and thrusters. This equation is a simple combination of these masses (4.1).

$$m_{Payload} = m_{Astro} + \frac{1}{9}(m_{Food} + m_{Water}) + m_{Equip} + m_{Thrust} + m_{IPV1S} \quad (4.1)$$

The mass of the various categories is determined by the volume of their respective compartments and either an estimated density (for example, the "density" per given volume of equipment, which may be estimated using data from prior launches) or an estimated mass. In the case of personnel, this value will be the average weight of a person times the number of crew members. Note that the fuel is not considered, as this is not a deliverable payload. In this case, the fuel will be a constraint of the IPV. Figure 4.1-1 is the planned layout for the two halves of the IPV.

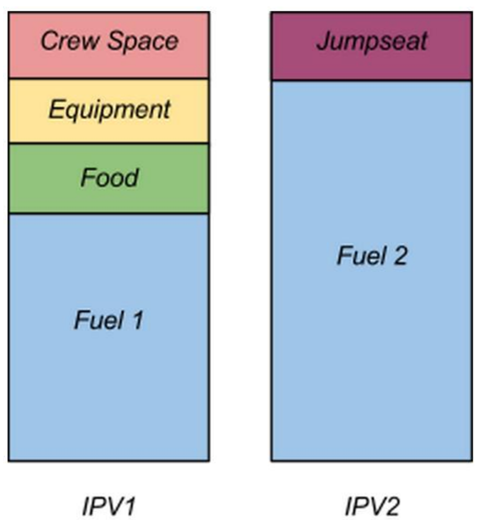


Figure 4.1-1: IPV1 and IPV2 cross-sectional models

The fuel used in this analysis does not diffuse through its tanks over time, but others like hydrogen fuel can. The IPV layout was designed to minimize losses throughout the mission, even though N_2O_4 and AZ_{50} do not. Since the IPV1 sits in orbit for up to a year before the IPV2 joins it, it was ideal to store more fuel in IPV2. Similarly, astronauts use up materials and can atrophy while in space. To minimize the manned travel time, a jumpseat was added to IPV2 to seat the astronauts for launch. When the two halves dock together, the astronauts would then enter the crew space for the remainder of the journey.

To calculate the payload, several simple mass equations based on the densities and volumes of various objects were used. Below are the calculations for the astronauts, food, water, and equipment taken to Mars. This model assumes equipment is shared among the astronauts.

$$m_{Astro} = n_{Astro} * m_{Person} \tag{4.2}$$

$$m_{Food} = n_{Astro} * \rho_{Food} * \pi * r^2 * h_{Food} \tag{4.3}$$

$$m_{Water} = n_{Astro} * \rho_{Food} * \pi * r^2 * h_{Water} \tag{4.4}$$

$$m_{Equipment} = \rho_{Equip} * \pi * r^2 * h_{Equip} \tag{4.5}$$

Part of the final payload also includes the empty vehicle mass that lands on the surface. This includes the structural mass of the IPV1 and the Thruster. The empty IPV1 mass takes the empty mass to surface area ratio from the Apollo Command Module (ACM) and applies it to the IPV1 and IPV2 structural masses. This provides a reasonable estimate for the structural weight needed for the IPV build.

$$RS = \frac{m_{ACM\,S}}{2\pi(r_{ACM}^2 + r_{ACM}h_{ACM})} \tag{4.6}$$

$$m_{IPV1\,S} = RS \cdot 2\pi\left[r^2 + r * \left(h_{Food} + h_{Water} + h_{Equip} + h_{fuel1}\right)\right] \tag{4.7}$$

$$m_{IPV2\,S} = RS \cdot 2\pi\left(r^2 + r\left(h_{Jump} + h_{fuel2}\right)\right) \tag{4.8}$$

4.1.1 Constraints

This subsystem is subject to similar dimensional and delta v constraints as those found in Section 3.1.1 above. The total available fuel stored in the two halves must be able to travel to Mars and perform a controlled burn with a soft landing on the Martian surface. As discussed in Section 3.1.1, the fuel needed for space maneuvers can be calculated using delta-v in the rocket equation. The IPV needs a delta-v of 4600 m/s to fly from low Earth orbit to low Mars orbit, and another 6800 m/s to softly land on the Martian surface. During the trip from Earth orbit to Mars orbit, the fuel tanks in IPV2 will run dry and can be detached from the IPV. This will decrease the mass of the vehicle and save fuel. This staging maneuver requires a recalculation of the rocket equation part way through the maneuver because of the sudden mass loss. Thrust and engine mass flow rate are both solely dependent on the engine specifications.

The delta-v calculations for the MTO and the TMI are shown in Equations 4.9 through 4.11, and represent the total fuel requirement of the IPV.

$$\Delta v_1 = \frac{T}{\dot{m}_{Engine}} * \ln\left(\frac{m_{Empty1}+m_{Fuel2}}{m_{Empty1}}\right) \tag{4.9}$$

$$\Delta v_2 = \frac{T}{\dot{m}_{Engine}} * \ln\left(\frac{m_{Empty2}+(1-FRR)*m_{Fuel1}}{(1-FRR)*m_{Empty2}}\right) \tag{4.10}$$

$$\Delta v_3 = \frac{T}{\dot{m}_{Engine}} * \ln\left(\frac{m_{Empty3}+FRR*m_{Fuel1}}{FRR*m_{Empty3}}\right) \tag{4.11}$$

In the above equations, the fuel remainder ratio (FRR) is the fraction of fuel1 that is used to land on the surface of Mars from low mars orbit. Δv_1 and Δv_2 together are the velocity changes needed to travel from orbit to orbit. The empty masses are simple sums shown below.

$$m_{Empty1} = m_{IPV1\,Full} + m_{Astronauts} + m_{Thruster} + m_{IPV2\,Empty} \tag{4.12}$$

$$m_{Empty2} = m_{IPV1\,Empty} + m_{Food} + m_{Water} + m_{Equipment} + m_{Astronauts} + m_{Thruster} \tag{4.13}$$

$$m_{Empty3} = m_{Objective\,Payload} \tag{4.14}$$

To calculate how much space will be needed for the mass of fuel, the densities of the fuel mixture was used. The Apollo Command Module used a fuel mixture of 1.9 parts dinitrogen tetroxide (N_2O_4) for every part Aerozine 50 (AZ_{50}) by mass for the AJ10-137 engine. The masses of fuel were calculated from these densities.

$$m_{fuel1} = \frac{1.9*\rho_{N_2O_4}+\rho_{AZ_{50}}}{1.9+1} * \pi * r^2 * h_{Fuel1} \tag{4.15}$$

$$m_{fuel2} = \frac{1.9*\rho_{N_2O_4}+\rho_{AZ_{50}}}{1.9+1} * \pi * r^2 * h_{Fuel2} \tag{4.16}$$

In order to minimize the OLB, the OLB payload should be equivalent on each of the two launches to get the two IPV halves into orbit. To do this, a constraint was made that forced the two halves on launch to be the same mass and roughly the same volume. Additionally, each half needed to be lighter than the Saturn V payload to ensure that the OLB would be able to lift the IPV halves into orbit. Height and radius restrictions were also placed on the IPV to ensure that the rocket would be realistically sized.

The various dimensional, mass, and numerical constraints on the design variables and overall masses are as follows:

$$2\,m \le r \le 5\,m$$
$$h_{Crew} \ge 3\,m$$
$$h_{Food} \ge 0\,m$$
$$h_{Water} \ge 0\,m$$
$$h_{Equipment} \ge 0\,m$$
$$h_{Fuel1} \ge 0.01\,m$$
$$h_{Jumpseat} \ge 1\,m$$
$$h_{Fuel2} \ge 0.01\,m$$
$$n_{Astro} \ge 1\,Person$$
$$n_{Astro} = integer$$
$$0.01 \le FRR \le 0.99$$
$$m_{IPV1\,Full} \le 122{,}700\,kg$$
$$m_{IPV2\,Full} \le 122{,}700\,kg$$

4.1.2 Design Variables and Parameters

Several of the variables seen previously were varied in order to discover the optimal solution. These include the radius of the IPV, the heights of each compartment, the number of astronauts, and the fuel remainder ratio. These nine quantities are highlighted in yellow in the following table.

Table 4.1.2-1: Results for IPV Subsystem Optimization using Apollo CSM Inputs

Objective Function			Design Variables			Addtl. Constraint Variables			Imports			Apollo CSM Constants		
mtot =	6019	*kg*	r_1 =	2	*m*	ΔvMT =	4600	*m/s*	mT_1 =	100	*kg*	CSM		
			h_1 =	3.8465	*m*	ΔvMS =	6800	*m/s*	n_1 =	1		L =	42.1	*m*
Intermediates			h_2 =	0.1366	*m*	FPP =	514.8	*kg*	T_1 =	91	*kN*	LST =	42.1	*m*
$mIPV_1$ =	2438	*kg*	h_3 =	0.0379	*m*	WPP =	476.28	*kg*	t_1 =	4115	*s*	R =	5.05	*m*
m_2 =	514.8	*kg*	h_4 =	0.1746	*m*	MLR =	1	*kg/kg*	t_2 =	2315	*s*	GVW =	2290000	*kg*
m_3 =	476	*kg*	h_5 =	7.7545	*m*	VLR =	1.3159	m^3/m^3	t_3 =	1633	*s*	EVW =	130000	*kg*
m_4 =	3291	*kg*	h_6 =	1	*m*	m_1tot =	122700	*kg*	mP =	18000	*kg*	STW =	130000	*kg*
m_5 =	115879	*kg*	h_7 =	8.0816	*m*	m_2tot =	122700	*kg*	ρN2O4 =	1442	kg/m^3	NFW =	2160000	*kg*
mnT_1 =	100	*kg*	Na =	1	*nauts*				ρAZ50 =	892	kg/m^3	tB =	165	*s*
$mIPV_2$ =	1853.1	*kg*	FRR =	0.4136	*kg/kg*	Intermediates			MRF_1 =	1.9	*kg/kg*	nT =	1	*thrusters*
m_7 =	120767	*kg*				Δv_1 =	2100.6	*m/s*	VRF_1 =	1.18	m^3/m^3			
ma =	80	*kg*				Δv_2 =	2499.4	*m/s*	RS_1 =	16.24	kg/m^2	AJ10-137		
						Δv_3 =	6800	*m/s*				L =		*m*
						mE_1 =	124633	*kg*	ρ_2 =	300	kg/m^3	M =		*kg*
						mE_2 =	54832	*kg*	ρ_3 =	1000	kg/m^3	mdot =	29.35	*kg/s*
						mE_3 =	6018.7	*kg*	ρ_4 =	1500	kg/m^3	T =		*kN*
									ρa =	80	*kg/naut*			

4.1.3 Summary Model

Maximize:

$$m_{Payload} = m_{Astro} + \frac{1}{9}(m_{Food} + m_{Water}) + m_{Equip} + m_{Thrust} + m_{IPV1\,S}$$

Subject To:

$g_1 = -r \leq -2\,m$ $g_2 = r \leq 5\,m$ $g_3 = -h_{Crew} \leq -3\,m$
$g_4 = -h_{Food} \leq 0\,m$ $g_5 = -h_{Water} \leq 0\,m$ $g_6 = -h_{Equip} \leq 0\,m$
$g_7 = -h_{Fuel1} \leq -0.01\,m$ $g_8 = -h_{Jump} \leq -1\,m$ $g_9 = -h_{Fuel2} \leq -0.01\,m$
$g_{10} = -n_{Astro} \leq -1\,Person$ $g_{11} = -FRR \leq -0.01$ $g_{12} = FRR \leq 0.99$
$g_{13} = m_{IPV1\,Full} \leq 122{,}700\,kg$ $g_{14} = m_{IPV2\,Full} \leq 122{,}700\,kg$
$g_{15} = n_{Astro} = integer$

Design Variables:

Radius of the IPV (r)
Height of the crew space (h1)
Height of the food compartment (h2)
Height of the water compartment (h3)
Height of the equipment compartment (h4)
Height of the fuel1 tank (h5)
Height of the jumpseat (h6)
Height of the fuel2 tank (h7)
Number of astronauts (Na)
Fuel Remainder Ratio (FRR)

4.2 Model Analysis

To ensure that the global optimum will be selected, monotonicity analysis was utilized to check if the problem was well constrained. The results are shown in Table 4.2-1. The monotonicity table shows that the problem is well bounded both above and below. It does also show that g1 and g3 thru g11 will be active. What this table does not account for is the integer constraint placed on the number of astronauts. This will force all of these variables to be less than their upper bound.

Table 4.2-1: Monotonicity Analysis for IPV Subsystem

	r	h1	h2	h3	h4	h5	h6	h7	Na	FRR
f	+	+	+	+	+	+	+	+	+	+
g1	-									
g2	+									
g3		-								
g4			-							
g5				-						
g6					-					
g7						-				
g8							-			
g9								-		
g10									-	
g11										-
g12										+
g13	+	+	+	+	+	+			+	
g14	+						+	+		

4.3 Optimization Study

The optimal solution was found using Microsoft Excel GRG Nonlinear Solver. The solution did not use the upper bounds for all of the variables due to the integer constraint. It was found that 3 astronauts would be optimal as four astronauts and all their equipment would exceed the lifting capacity of the Saturn V. The IPV was then designed around the equipment and supplies for 3 astronauts. The analysis was given several different starting points and arrived at the same solution each time.

Since many of the equations used were linear combinations of the design variables, the optimum would be on the boundaries if there was not an integer constraint involved. Because four astronauts and all their supplies would exceed the lifting capacity of the Saturn V, the solver had to use 3 astronauts and then optimize the geometry to accommodate.

4.4 Parametric Study

To verify the solution obtained, several of the variables were changed and their effects on different variables were measured. Figures # and # show how the solution changes when a certain number of astronauts are forced. The payloads of both the IPV and OLB increase linearly with the number of astronauts. The height also increases linearly, but the radius stays constant at its lower bound. It is suspected that this is because of how the structural mass is calculated. When adding volume to a cylinder, adding height increases the surface area linearly while increasing the radius increases the surface area exponentially. Because the structural mass is dependent on surface area instead of volume, the solver chose to increase the height to compensate for the needed volume and keep the radius at its lowest possible value.

Figure 4.4-3 is a display of how the IPV and OLB payloads increase as the radius increases with a constant astronaut count. The payload mass increases proportional to the square of the radius. This further supports the idea that the equation that governs the structural mass greatly influences the ideal IPV geometry and fuel usage. The added structural mass from the increased surface area also requires more fuel to complete the journey to Mars.

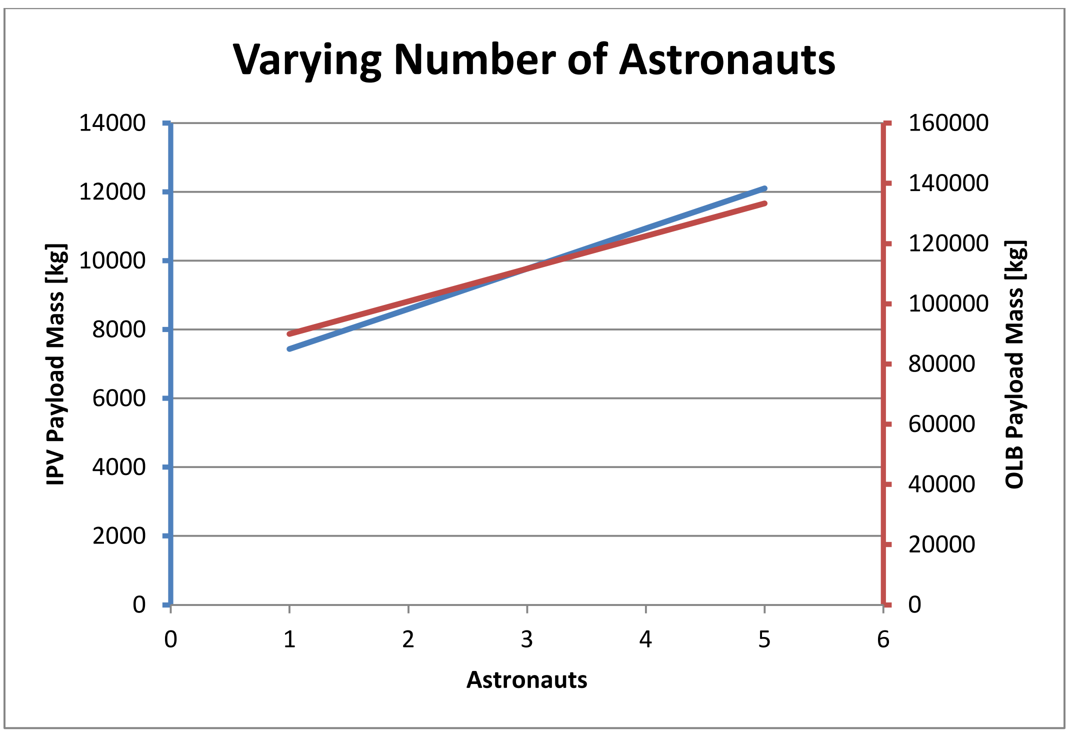


Figure 4.4-1: Resulting IPV and OLB Payloads when Varying Number of Astronauts

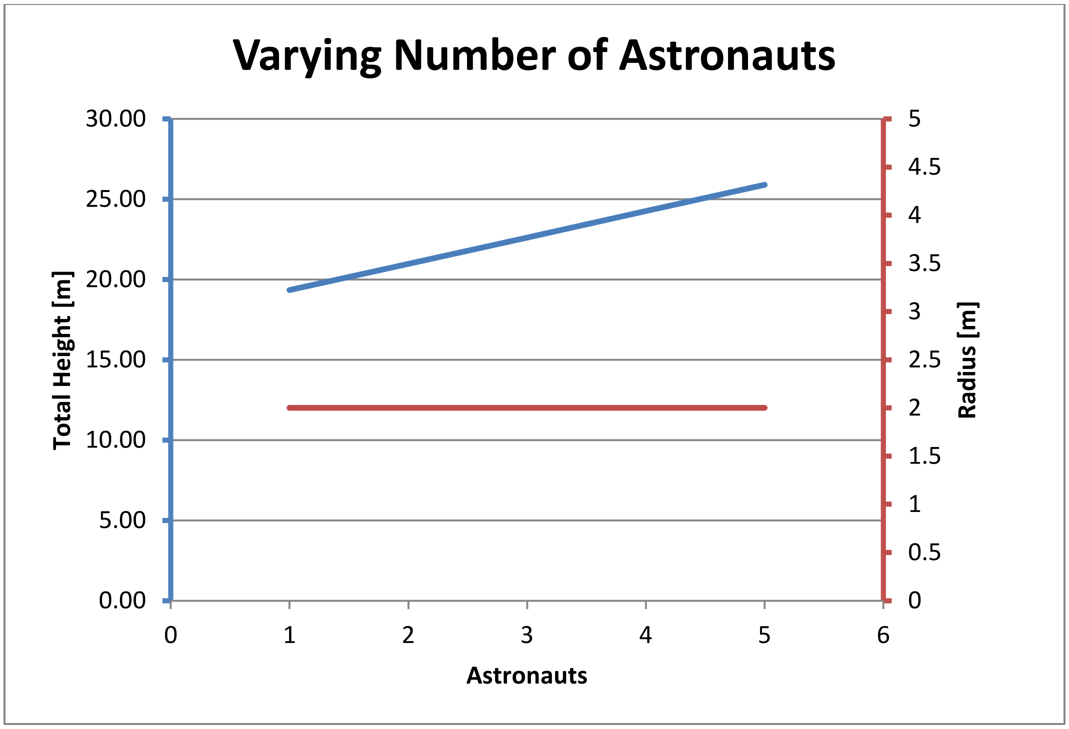


Figure 4.4-2: Resulting Height and Radius when Varying Number of Astronauts

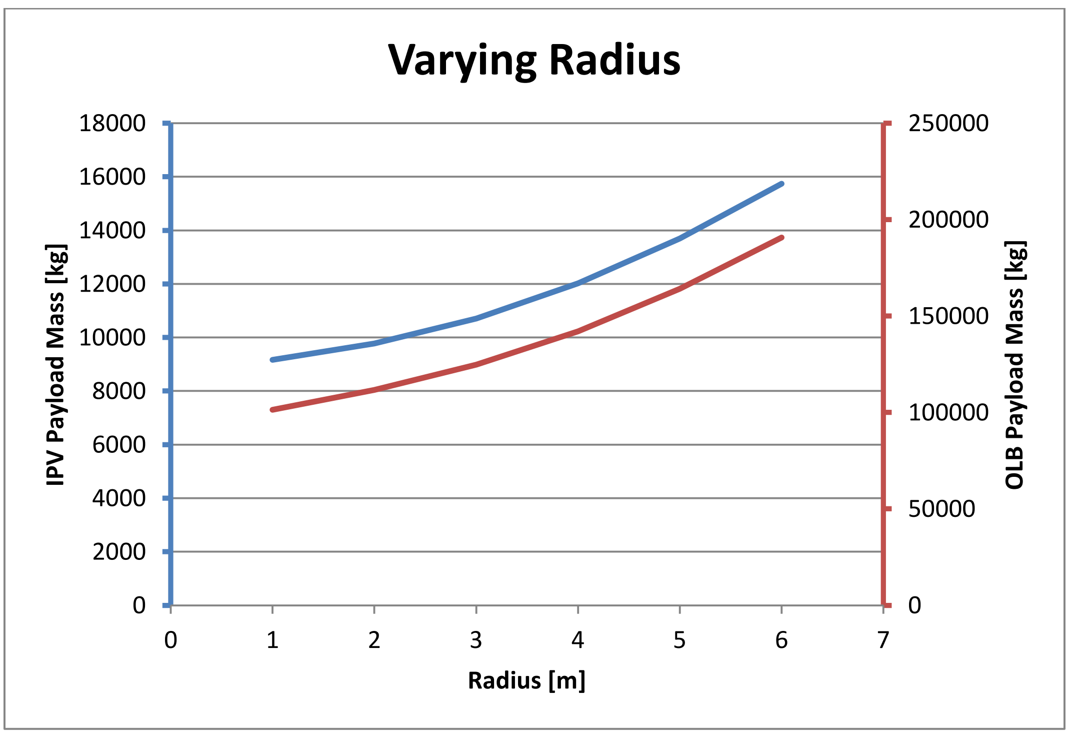


Figure 4.4-3: Resulting IPV and OLB Payloads when Varying Radius

4.5 Discussion of Results

This model proves that using existing technology, a vehicle could be made that would successfully travel from earth to mars to start a colony. The mass of the designed IPV does not exceed the lifting capacity of the largest successful rocket launched. The maximum people this design can send is three people, but this does use some generalizing assumptions about the rocket structure and the materials brought. The results are promising for later iterations of the project when it will be combined with the OLB and PSL subsystems.

5. PROACTIVE SUPPLY LAUNCH (PSL) SUBSYSTEM

5.1 Mathematical Model

The objective of the PSL subsystem is to minimize the number of resupply launches. Replacements for each assembly must be delivered in a timely manner before failure occurs. It is imperative to optimally pack components into the fewest number of landers possible. Matrix $\bar{X}$ contains the number of spare assemblies with respect to each lander. Rows of $\bar{X}$ are equivalent to the number of landers available (I). Columns represent the number of critical assemblies (N) for each resupply launch.

$$\min = \sum_{i=1}^{I} \bar{X}$$

$\bar{X}$ row and column distribution:

$$\bar{X} = \begin{bmatrix} x_i^n & \cdots & x_i^N \\ \vdots & \ddots & \vdots \\ x_I^n & \cdots & x_I^N \end{bmatrix}$$

Row distribution:

$$[x_i^n \quad \cdots \quad x_i^N] = [x_{OG} \quad x_{CDR} \quad x_{CCA} \quad x_{UP} \quad x_{WP} \quad x_{CO2} \quad x_{ISRU} \;]$$

5.1.1 Constraints

The PSL subsystem is subject to the dimensional constraints generated by the IPV subsystem. It was determined that the maximum allowable mass was 9770 kg. The maximum volume was 79.87 m^3.

IPV Mass Constraint:

$$\sum_{n=1}^{N} x_i^n m^n \;\le\; M_{IPV} = 9770\,kg$$

$$\forall n \in \{1 \ldots, N\},$$

$$\forall i \in \{1 \ldots, I\},$$

IPV Volumetric Constraint:

$$\sum_{n=1}^{N} x_i^n V^n \leq V_{IPV} = 79.87m^3$$

$$\forall n \in \{1 \dots, N\},$$

$$\forall i \in \{1 \dots, I\},$$

Component Failure Constraint:

The component failure constraint ensures that each assembly will be replaced before failure. The Mars colony will start with a full set of the 7 critical assemblies. MTBF_0 is the average working life of each component. The MTBF of each of the assemblies is listed in Section 5.3.2. Shown below is the mathematical model of the constraint:

$$MTBF^n x_0^n + \sum_{t=1}^{T} MTBF^n x_t^n - MTBF^n t > 0$$

$$\forall n \in \{1 \dots, N\},$$

$$\forall i \in \{1 \dots, I\},$$

$$\forall t \in \{1 \dots, T\},$$

A graphical representation of the goal of the component failure constraint is shown in Figure 5.1.1-1 below. If the operational life of any component reaches zero it could have catastrophic results. It would put in jeopardy the life of the astronauts and it would make the Mars base inhabitable.

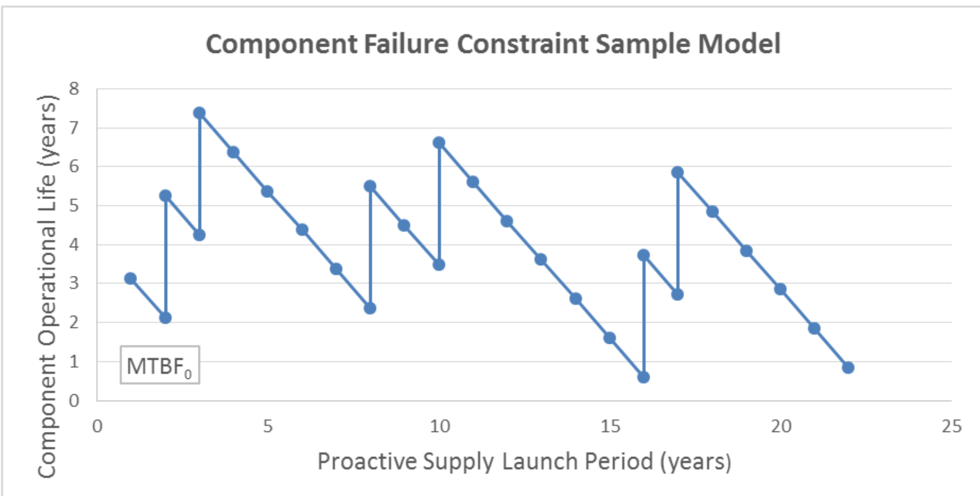


Figure 5.1.1-1: Example PSL Failure Constraint

5.1.2 Design Variables and Parameters

The critical assemblies were chosen to ensure crew health and safety. Maintaining a habitable atmosphere in the Mars base involves providing proper atmospheric components in precise quantities necessary to sustain life. Earth atmosphere is composed of 78.1% nitrogen (N_2), 21.0% oxygen (O_2), and other miscellaneous gases including carbon dioxide (CO_2) and water vapor.

The oxygen generation assembly has several functions. Oxygen must be at a suitable partial pressure for the metabolic system to function correctly. An oxygen rich atmosphere can lead to inflammation of the lungs and hyperoxia. Symptoms of hyperoxia are convulsions, fainting, and dizziness. An oxygen deprived atmosphere can cause sleepiness, headache, and also loss of consciousness. Low levels of oxygen can also lead to hypoxia. Hypoxia seriously impairs the brain. Carbon dioxide (CO_2) is a product of respiration. Concentration of CO_2 will increase sharply in confined areas. There is no minimum CO_2 requirement to maintain a habitable atmosphere. However, high levels of carbon dioxide will lead to headaches, nausea, rapid breathing, and increased heart rate. Both the carbon dioxide removal and reduction assemblies will maintain an ideal 0.04 % CO_2 content in the Mars colony atmosphere.

The primary function of the common cabin assembly is humidity control. Humidity is the amount of water vapor in the atmosphere relative to the maximum amount of water vapor the atmosphere can hold at a given temperature (Human Integration Design Handbook, 2010). Low humidity levels may cause dry eyes, skin, nose, and throat which in turn can lead to respiratory infections. High humidity levels can lead to microbial and fungal growth. Humidity levels must be between maintained at 30% - 50% over a 24 hour time span.

Water is critical for extraterrestrial colonization. The water processor assembly addresses the need to provide safe, useful, and palatable water. Any water used by the crew for drinking or hygiene must be potable, to ensure crew health. The average crewmember consumes 2.8 liters of water per day on average. The urine processor assembly will ensure that body waste is managed efficiently. Urine can be recycled to provide high-quality potable water in water deprived zones.

Finally, the ISRU assembly contains other miscellaneous critical components to ensure a habitable internal atmosphere. ISRU functions include: temperature and ventilation monitoring and control.

Listed below are the dimensional parameters of the 7 critical assemblies. The article *An Independent Assessment of the Technical Feasibility of the Mars One Mission Plan* was referenced to generate the values.

Table 5.1.2-1: Critical System Parameters

m_{OG}	223.13	*kg*
m_{CDR}	156.32	*kg*
m_{CCA}	100.91	*kg*
m_{UP}	244.67	*kg*
m_{WP}	620.85	*kg*
m_{CO2}	219.49	*kg*
m_{ISRU}	220.82	*kg*
V_{OG}	0.2542	m^3
V_{CDR}	0.4239	m^3
V_{CCA}	0.6097	m^3
V_{UP}	0.4837	m^3
V_{WP}	0.7537	m^3
V_{CO2}	0.6812	m^3
V_{ISRU}	1.1986	m^3
$MTBF_{OG}$	5.419977169	*years*
$MTBF_{CDR}$	3.755707763	*years*
$MTBF_{CCA}$	3.755707763	*years*
$MTBF_{UP}$	3.12	*years*
$MTBF_{WP}$	2.92	*years*
$MTBF_{CO2}$	5.707762557	*years*
$MTBF_{ISRU}$	7.610353881	*years*
M_{IPV}	9770	*kg*
V_{IPV}	79.87	m^3

5.1.3 Summary Model

Minimize:

Assembly distribution$= \sum_{i=1}^{I} \bar{X}$

$\forall i \in \{1 \ldots, I\}$,

Subject To:

IPV Mass Constraint:

$g_1 = \sum_{n=1}^{N} x_i^n m^n \;\; \leq \; M_{IPV}$

$\forall n \in \{1 \ldots, N\}$,

$\forall i \in \{1 \ldots, I\}$,

IPV Volumetric Constraint:

$g_2 = \sum_{n=1}^{N} x_i^n V^n \;\; \leq \; V_{IPV}$

$\forall n \in \{1 \ldots, N\}$,

$\forall i \in \{1 \ldots, I\}$,

Component Failure Constraint:

$g_3 = -(MTBF^n x_0^n + \sum_{t=1}^{T} MTBF^n x_t^n - MTBF^n t\,) < 0$

$\forall n \in \{1 \ldots, N\}$,

$\forall i \in \{1 \ldots, I\}$,

$\forall t \in \{1 \ldots, T\}$,

5.2 Model Analysis

The PSL subsystem initially consisted of 86 components. In order to simplify the system, components were combined to form 7 critical assemblies. The total mass and volume of each assembly was calculated using the dimensional properties of the original 86 components. The PSL subsystem was designed to allow for daily launches if needed. 7665 total landers were originally available. The system was simplified to annual deployments meaning 21 landers were available. The number of variables was significantly reduced from 659190 to 147.

The original PSL subsystem had too many variables to check all cases with loops. Instead a genetic algorithm (GA) was used to solve the PSL subsystem. The *ga* function will output an answer containing integers. The lander cargo cannot be composed of fractional assemblies. Additionally, it should be noted that all constraints are linear inequality constraints therefore the PSL subsystem meets the monotonicity criteria. A lower bound of 0 was chosen to ensure no component is negative.

5.3 Optimization Study

The solution for the PSL subsystem is shown below in Table 5.3-1. Only 15 of 21 available landers were used. The PSL subsystem was optimized.

Table 5.3-1: PSL Optimal Launch Plan

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
0	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	0	0	1	1	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	1	0	0	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0
1	1	0	0	0	0	1	0	1	0	0	0	0	0	1	1	0	0	0	0	0
0	1	1	0	0	0	0	1	0	0	0	1	0	0	1	0	0	1	0	1	0
1	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0

5.4 Discussion of Results

A breakdown of each of the payloads is shown in Figure 5.4-1. The maximum payload mass required was 1021.84 kg. It should be noted that none of the payload masses are in danger of reaching the maximum allowable payload mass calculated in the IPV subsystem.

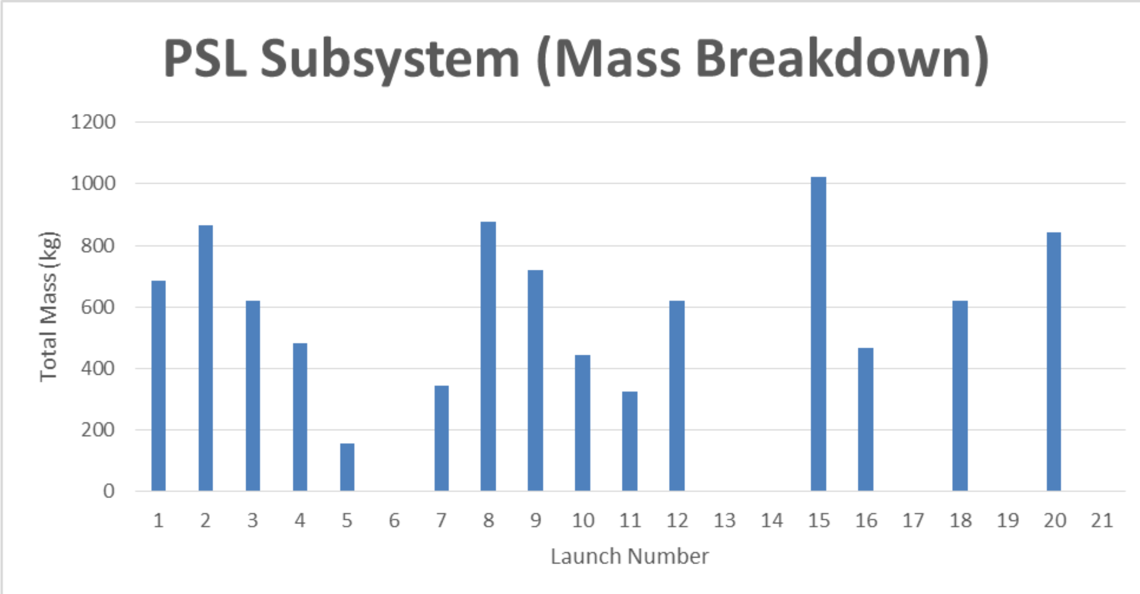


Figure 5.4-1: PSL Payload Characteristics

A graphical representation of the PSL payload breakdown is shown in Figure 5.4-2. This figure highlights the actual subsystems that would be sent in each payload. The urine processor assembly had the highest need for spares. The solution is congruent with the assemblies' MTBF values as the urine processor has the lowest working life with an average failure time of 2.92 years. The maximum volume needed 2.3635 m^3.

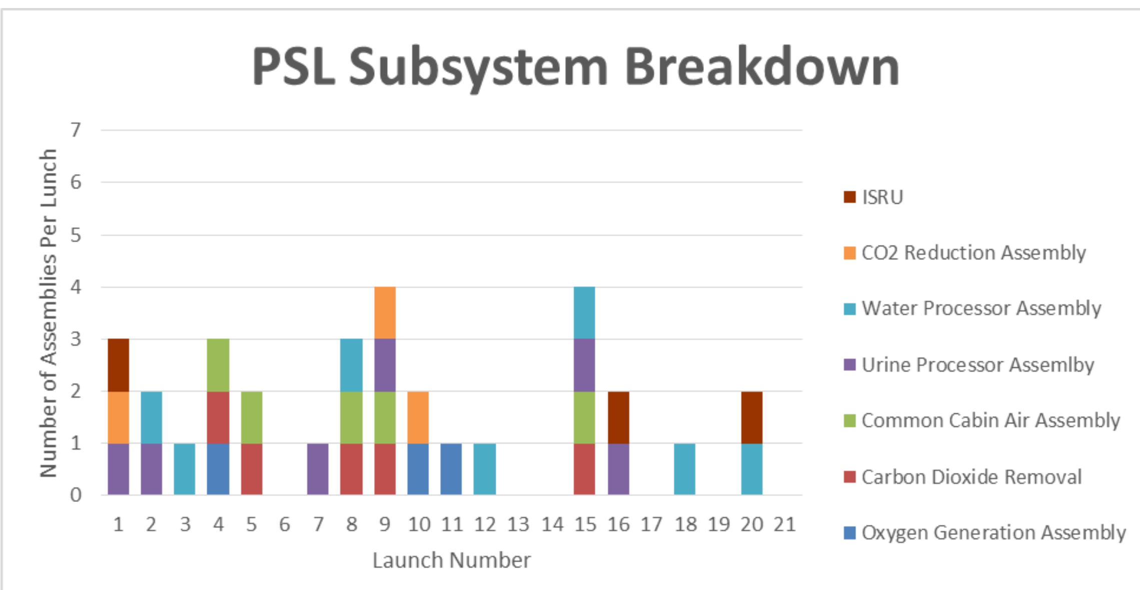


Figure 5.4-2: PSL Payloads by Replacement Assembly

6. SYSTEM INTEGRATION STUDY

Although each subsystem was able to identify an optimal solution, these combined optima do not represent the solution for an actual mission to mars. This is because the studies heretofore conducted were performed in order to validate the subsystem models. In reality, these three subsystems have several interactive constraints that must be satisfied in turn. The optimal IPV payload mass must not violate the lifting capacity of the OLB, and both the payload mass and volumetric capacity of the IPV must be able to accommodate the PSL. An illustration of these constraint relations are shown in Figure 6-1. Since the simultaneous solution of all subsystems was infeasible, an iterative approach was utilized instead.

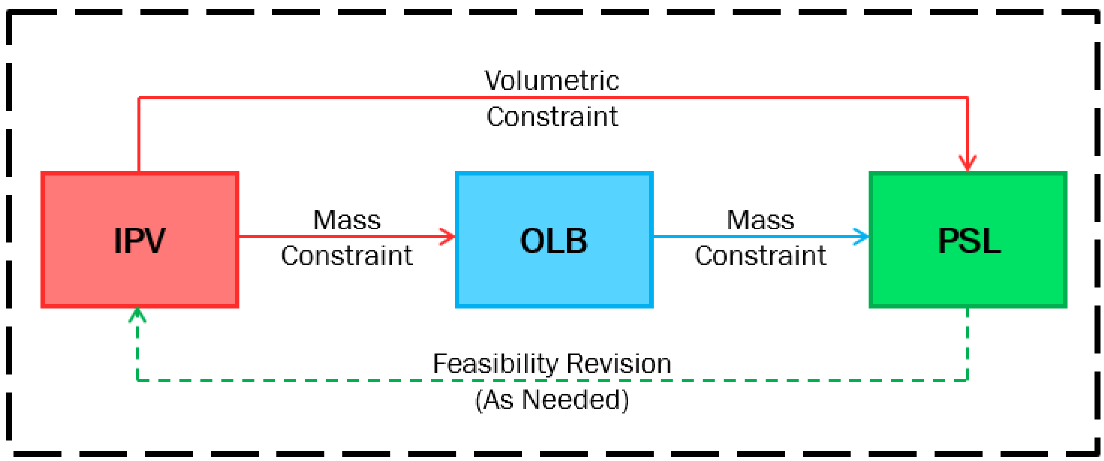


Figure 6-1: Integrated System Flowchart

In order to implement the integrated system effectively it was necessary to select an overall objective function. The design team chose to maximize the number of astronauts involved in the mission, as this represents a more ambitious engineering challenge. The IPV was selected as the starting point since it already included the number of astronauts.

The IPV subsystem was solved as described in Section 4, utilizing the Excel GRG nonlinear solver. While the integrated IPV subsystem model was nearly identical to the isolated subsystem model, there was one key difference: instead of allowing the solver to vary the number of astronauts, the optimal solution for multiple cases was found by manually varying the number of astronauts between one and four. This was useful because it removed the numerical instabilities that had been introduced by the integer constraint. With only four cases to solve, manual manipulation was not a time consuming process for the IPV. The optimal results for each of these cases - including payload mass, total mass, overall height, and all dimensional values - were recorded and saved separately so that they could be easily loaded into MATLAB.

Once all IPV data had been collected, the OLB subsystem then had to be solved. As before, the integer constraints (the number of thrusters per stage) posed the most considerable challenges with respect to selecting a solution method, and measures were once again taken to eliminate them. The first logical choice was to simply remove these constraints from the solver and manually manipulate them, thus reducing the nonlinearity issues. Unfortunately, while the four IPV cases could easily be solved manually, each of those four cases had a potential 108 booster configurations. Due to the infeasibility of solving a combined 432 cases, an alternative method was needed.

The MATLAB *fmincon* solver proved to be the ideal solver choice for this situation. The implementation of for loops meant that the integer constraints could be automatically inputted to the solver, which would then in turn generate a solution for each case. The required payload for each number of astronauts was also easily imported from the IPV results file. The SQP optimization algorithm was selected, as the included approximate Hessian improved the stability. The optimal OLB results for each combination of astronauts and thrusters were stored in a series of arrays. If the solver terminated unfavorably (as indicated by its exit criteria) then the dimensions were set to zero meters and the objective value was set to an impossibly high value of one million metric tons. The objective function array - which stored the total OLB mass values - was searched for the minimum once all solutions for that number of astronauts had been obtained. The indices for that minima could then be used to pull the associated dimensions of the booster stages, which were then plotted.

When the OLB solutions were all obtained, the results were compared. The overall optimal solution at this stage was the maximum number of astronauts that could still be carried by an optimal OLB. Once this value was determined, the associated payload and volume for the current optimal solution was passed to the PSL for evaluation. The genetic algorithm would then be applied as described in Section 6. If a launch plan that met the needs of the astronauts for 22 years could be constructed then the system model was considered optimized. If not, then the number of astronauts would be reduced by one and the associated configuration would then be subjected to the PSL solver again.

7. DISCUSSION OF INTEGRATED SYSTEM RESULTS

The IPV results for 1 through 4 astronauts are shown in Table 7-1 below. Several trends are noticeable here, including the relatively linear increase in height, volume, and payload as the number of astronauts increases. The most notable trend is the proportion between payload mass and the "half-mass" (or mass of the heaviest IPV half). While the payload increased by an average of roughly 1.2 metric tons per additional astronaut, the associated half mass increased by 11 metric tons. Accordingly, much larger IPV halves would need to be built in order to accomplish the mission.

Table 7-1: Geometry and Mass of the IPV Subsystem for All Cases

Parameter	IPV Case			
Astronauts (#)	1	2	3	4
Radius (m)	2.0	2.0	2.0	2.0
Height (m)	19.3	21.0	22.6	24.3
Volume (m^3)	75.5	77.7	79.8	82.1
Payload (metric ton)	7.4	8.6	9.7	10.9
Half-Mass (metric ton)	90	101	112	123

The half-mass values indicate that the system optimization is feasible for at least one case; a mass of 90 metric tons represents only 75% of the mass already delivered to LEO by the Saturn V. However, at four astronauts the mass of the IPV reaches 123 metric tons, which exceeds the Saturn V's maximum payload by 5 metric tons. Due to the tyranny of the rocket equation, this discrepancy suggested that a crew of four astronauts may be in excess of the allowable maximum OLB capacity. Still, it was unclear at this phase whether that payload exceeded the feasible region of the design space. Accordingly, all four cases were considered in the OLB solution phase.

The OLB subsystem was solved for each of the four IPV cases. These scenarios resulted in feasible solutions for up to three astronauts. Four astronauts exceeded the OLB constraints as predicted based on the IPV data. Accordingly, no plot or results were generated for the four-astronaut case. The results are shown in Figures 7-1 through 7-3, each of which displays a visual representation of the OLB in black with the Saturn V in blue. The title of each plot also contains the general properties of the associated OLB. Note that the plots only show the height of the stages, with the staging cones (shown as the narrow bands or tapered sections connecting the stages) considered as a constant height for both the OLB and Saturn V. The axes are both in meters.

A visual comparison of all the boosters and the Saturn V is shown in Figure 7-4. Several interesting conclusions can be drawn from this comparison. First and foremost is that the optimal solution favored a tapered second stage, while the Saturn V only tapered down on the third stage. In general, the OLB had a larger first stage than the Saturn V, with a smaller second and third stage. This may be due to the structural ratio predictions, which may not accurately predict the structural mass of the stages as they change significantly in radius or diameter. Finally, it should be noted that the optimal OLB for three astronauts is both taller and heavier than the associated Saturn V. The structural ratio may once again play a role, but in a different way; the structural ratio of the lower stage is much higher than that of the upper stage, which could lead to the inflated weight as the size increases.

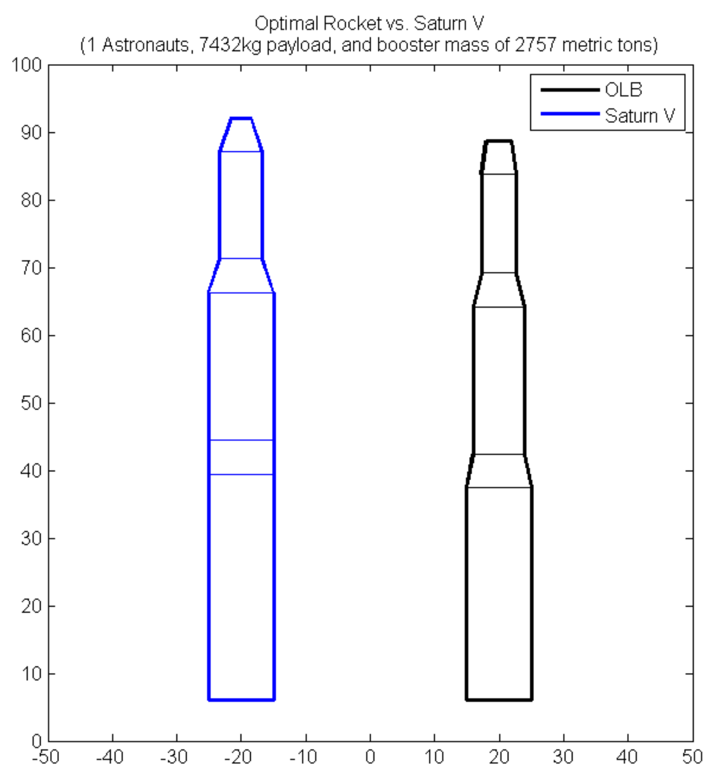


Figure 7-1: Single Astronaut OLB

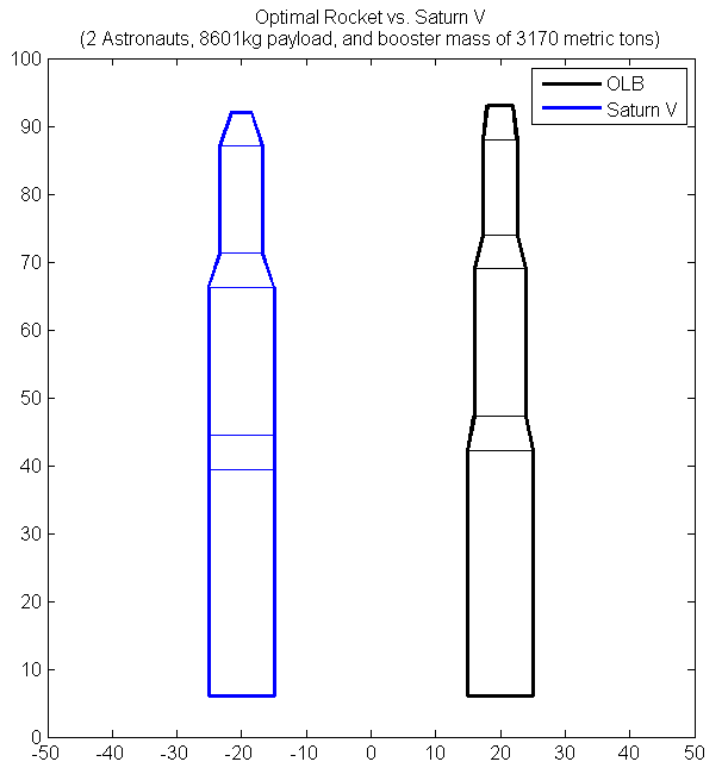


Figure 7-2: Two-Astronaut OLB

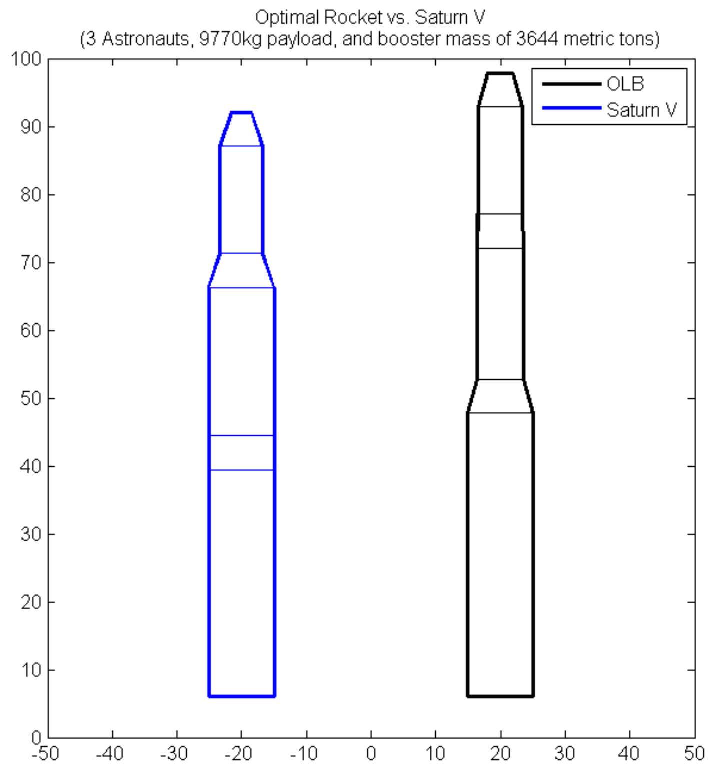


Figure 7-3: Three-Astronaut OLB

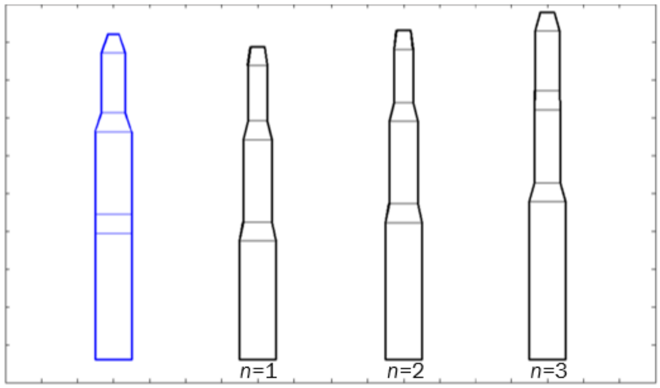


Figure 7-4: Geometries for n Astronauts Compared to the Saturn V

The visual similarity with the Saturn V is notable, which is expected given the magnitude of the proposed mission. The payload volume and mass constraints were passed to the PSL subsystem, which was solved as described in Section 5. The results are shown in Section 5.4. The fact that several of the supply launches did not need to be utilized and the payload and volume constraints were not active at the solution poses several interesting possibilities for the resupply plan. On one hand, these launches could easily be performed with smaller and cheaper rockets than the OLB. It would also be possible to send additional cargo launches and increase the size of the colony, or perhaps start new colonies at several different locations across the planet. The final option would be to condense the launch plan to a smaller time window.

The final results paint a picture of the final mission that could be performed. First, the OLB lifts the two halves of the IPV into earth orbit. The OLB is the largest and most powerful rocket ever built. Taller and heavier than the Saturn V, it could lift over one-third of the International Space Station into orbit in a single flight.

The IPV halves dock together and ferry the astronauts to the surface of Mars over the course of roughly eight months, staging twice in interplanetary space to shed extra mass. This state-of-the-art capsule carries all the fuel, food, water, and equipment necessary to survive the journey.

The crew lands near a pre-delivered supply drop and begin to set up their new home. The IPV continues to serve as a dwelling, generator, gym, and water treatment/waste processing facility. New shipments arrive over the course of 22 years, delivering backup and replacement systems before the originals fail. The astronauts continue to build and expand their colony, reaching a point of self-sufficiency before the last supply launch is delivered. Their mission represents the greatest scientific and engineering accomplishment in the history of the human race, and establishes our first real presence among the stars.

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A1. INITIAL OLB MODEL

The original model for the OLB consisted of only two stages, which proved to be inaccurate when attempting to match the Saturn V. This model was coded and solved via the GRG Nonlinear Excel solver. Input values in the form of a 118,000 kg payload and using five thrusters for each stage were used. This case was considered in an attempt to locate possible local minima. The results of this run are shown in Figure A1-1. The answer report is shown in Figure A2-2.

Objective Function		
mtot =	3633837	*kg*
Intermediates		
mf_1 =	2940439	*kg*
ms_1 =	106260.09	*kg*
mnT_1 =	41765	*kg*
mf_2 =	502719	*kg*
ms_2 =	33708	*kg*
mnT_2 =	8945	*kg*

Design Variables		
h_1 =	41.32701	*m*
r_1 =	5.05	*m*
h_2 =	24	*m*
r_2 =	5.05	*m*

Addtl. Constraint Variables		
Δv =	9000	*m/s*
Intermediates		
Δv_1 =	3979.318	*m/s*
Δv_2 =	5020.682	*m/s*
mE_1 =	811397	*kg*
mE_2 =	160652.7	*kg*

Imports		
mT_1 =	8353	*kg*
mT_2 =	1789	*kg*
n_1 =	5	
n_2 =	5	
T_1 =	34020	
T_2 =	4400	
t_1 =	225	
t_2 =	405	
mP =	118000	*kg*
ρLOX =	1141	kg/m^3
ρLH2 =	70.85	kg/m^3
ρRP1 =	915	kg/m^3
MRF_1 =	2.27	*kg/kg*
MRF_2 =	5.50	*kg/kg*
VRF_1 =	1.82	m^3/m^3
VRF_2 =	0.34	m^3/m^3
RS_1 =	72.21	kg/m^2
RS_2 =	36.57	kg/m^2

Figure A1-1: OLB Subsystem Case Study

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
C9	mf_2 =	502719	C9>=1	Not Binding	502718
C6	mf_1 =	2940439	C6>=1	Not Binding	2940438
K3	Δv =	9000	K3>=9000	Binding	0
G3	h_1 =	41.32701458	G3<=42	Not Binding	0.672985417
G3	h_1 =	41.32701458	G3>=1	Not Binding	40.32701458
G4	r_1 =	5.05	G4<=5.05	Binding	0
G4	r_1 =	5.05	G4>=1	Not Binding	4.05
G5	h_2 =	24	G5<=24	Binding	0
G5	h_2 =	24	G5>=1	Not Binding	23
G6	r_2 =	5.05	G6<=5.05	Binding	0
G6	r_2 =	5.05	G6>=1	Not Binding	4.05

Figure A2-2: OLB Case Study Answer Report - Constraints

The results of this case study are positive. The OLB is able to match the lifting capacity of the Saturn V using only two stages instead of three. The delta-v requirement is bound at 9 km/s as expected, since it represents the main constraint in nearly every rocketry scenario. The radii of both stages are equivalent, indicating that the optimal mass structure has a larger aspect ratio (this boundary is not increased due to the increasing limitations of drag on higher-profile vehicles). The height of the second stage is maximized while the first stage is not. This reflects the relative efficiency between the first and second stages - the first stage consumes fuel more quickly to achieve a comparable thrust. A table showing a comparison between the OLB case and the Saturn V is shown in Table A1-1.

Table A1-1: OLB Case v. Saturn V Comparison

Value	Saturn V	OLB	Net % Change
Height of All Stages	85.7 m	78.6 m	- 8.2%
Maximum Diameter	10.1 m	10.1 m	0.0%
EVM	183600 kg	193360 kg	+ 5.3%
GVM	2909200 kg	3720486 kg	+ 27.9%
Percent Fuel by Mass	94.1%	91.8%	- 2.3%
Payload to LEO	118,000 kg	118,000 kg	0.0%

Overall, the results of this comparison are less than ideal. While the overall height of the OLB is less than the Saturn V with a slightly higher empty mass, the fuel consumption necessary is enormously higher as shown by both the GVM and the percent fuel by mass. This discrepancy may be attributed in part to the approximations used, but the overwhelming jump in fuel is too excessive to ignore. While the resulting OLB is able to achieve LEO with the required payload it is definitively and significantly less efficient than the Saturn V.

The addition of a third stage was a logical and sensible next step to address this discrepancy. In order to modify the model, two new design variables were defined:

$$h_3 \equiv \text{height of stage 3}$$
$$r_3 \equiv \text{radius of stage 3}$$

The updated constraints for delta-v and the dimensions were added, and the model was revised and re-run with the three-stage configuration as discussed in Section 3.

A2. MATLAB CODE – OLB SOLVER AND PLOTTER

```
function Mission2Mars
clc
clear
%% Solve for the ideal rocket
% IPV Maximum Half-Masses
mIPV = [89999, 100837, 111675, 122513];
```

```
% IPV Dimensions
rIPV = [2.00, 2.00, 2.00, 2.00];
hIPV = [12.0, 12.8, 13.7, 14.6];

% IPV Deliverable Payload
PIPV = [7432, 8601, 9770, 10939];

% Solution Array
optimal=[];

for i = 1:4
for j = 1:6
for k = 1:6
for l = 1:3
    clc
    status=['Solving for ' int2str(i) ' astronauts and the following
'...
            'configuration:\n '...
             int2str(j) ' stage 1 engines\n '...
             int2str(k) ' stage 2 engines\n '...
             int2str(l) ' stage 3 engines\n '...
            ];
    fprintf(status);

    % Define the Initial Guess
    r0 = [1,1,1];
    h0 = [5,5,5];

    % Define the Number of Engines
    n0 = [j,k,l];

    % Define the Necessary Payload Mass
    mP0= mIPV(i);

    % Run the solver and Collect the Results
    [rv,hv,fv]=MARS_OLBv2(r0,h0,n0,mP0);

    for z = 1:3
        r{z}(j,k,l) = rv(z);
        h{z}(j,k,l) = hv(z);
    end
        f(j,k,l) = fv;
end
end
end

%% Identify the optimal solution for this number of astronauts

[~,I] = min(f(:));
[I1,I2,I3] = ind2sub(size(f),I);

    fopt=f(I1,I2,I3);
    nopt=[I1,I2,I3];
```

```
for z = 1:3
    ropt(z)=r{z}(I1,I2,I3);
    hopt(z)=h{z}(I1,I2,I3);
end

optimal=[optimal;i,fopt,ropt,hopt,nopt];

%% Plot the solution rocket

if fopt < 1e12
figure('position',[150 150 700 700])

hp=[6];
rp=[];

for y=1:3
    hp=[hp,hp(end)+hopt(y)]; hp=[hp,hp(end)+5];
    rp=[rp,ropt(y),ropt(y)];
end

rp=[rp,2];
h1 = plot(rp+20,hp,'-k','linewidth',2);
hold on
plot(-rp+20,hp,'-k','linewidth',2);

for y=1:length(hp)
    if y==1 || y==7
        plot([-rp(y),rp(y)]+20,[hp(y),hp(y)],'-k','linewidth',2)
    else
        plot([-rp(y),rp(y)]+20,[hp(y),hp(y)],'-k','linewidth',1)
    end
end

% Plot the Saturn V rocket
hs=[33.46, 21.8, 15.8];
rs=[5.05, 5.05, 3.3];

hp=[6];
rp=[];

for y=1:3
    hp=[hp,hp(end)+hs(y)]; hp=[hp,hp(end)+5];
    rp=[rp,rs(y),rs(y)];
end

rp=[rp,1.5];
h2 = plot(rp-20,hp,'-b','linewidth',2);
hold on
plot(-rp-20,hp,'-b','linewidth',2);

for y=1:length(hp)
    if y==1 || y==7
        plot([-rp(y),rp(y)]-20,[hp(y),hp(y)],'-b','linewidth',2)
    else
```

```
        plot([-rp(y),rp(y)]-20,[hp(y),hp(y)],'-b','linewidth',1)
    end
end
axis([-50 50 0 100])
legend([h1 h2],{'OLB','Saturn V'})

% Label the Figure
ptitle=({['Optimal Rocket vs. Saturn V'] ; ['(' int2str(i) '
Astronauts, '...
        int2str(PIPV(i)) 'kg payload, and booster mass of '
int2str(fopt/1000) ' metric tons)']});
title(ptitle)
pause(0.5)
end
end
```

```
function [r,h,f]=MARS_OLBv2(r0,h0,n0,mP0)
% This solver finds the optimal solution for the rocket given a payload
and
% the total number of thrusters

% Standalone Operation Mode
solo = 'false'; %True or False
true = 'true';

if strcmp(solo,true) == 1
    r0 = [5,4,3];
    h0 = [30,20,10];
    n0 = [5,6,3];
    mP0= 100837;
end

% Linear equality constraints of the form Ax = b
Aeq = [];
beq = [];

% Linear inequality constraints of the form Ax <= b
Ain = [];
bin = [];

% Nonlinear constraints are defined by subfunction nonlcon

% Bounds on the design variables
lb = [1.00*ones(1,3),1.00,0.10,0.10,n0,mP0];
ub = [5.05*ones(1,3),42.0,25.0,15.8,n0,mP0];

% Implement the solver

options = optimoptions('fmincon','Algorithm','sqp','MaxFunEvals',1000);

x0 = [r0,h0,n0,mP0];
[x,fval,exit] =
fmincon(@fun,x0,Ain,bin,Aeq,beq,lb,ub,@nonlcon,options);
```

```
r = x(1:3);
h = x(4:6);
n = x(7:9);
mP= x(10);
f=fval;

if sum(n-n0) ~= 0 && exit > 1
    error('The OLB solver changed the number of engines!')
end

if mP-mP0 ~= 0 && exit > 1
    error('The OLB solver changed the payload mass!')
end

if exit < 1
    r = 0*x(1:3);
    h = 0*x(4:6);
    f = 1e12;
end
end

function f = fun(x)
% % Rename variables for simplicity
% r1 = x(1); r2 = x(2); r3 = x(3);
% h1 = x(4); h2 = x(5); h3 = x(6);

r = x(1:3);
h = x(4:6);
n = x(7:9);
mP= x(10);

% Rocket Masses
[mF,mS,mT]=rocketmass(r,h,n);

% Objective Function
f = sum(mF+mS+mT);
end

function [c,ceq]=nonlcon(x)
% % Rename variables for simplicity
% r1 = x(1); r2 = x(2); r3 = x(3);
% h1 = x(4); h2 = x(5); h3 = x(6);

r = x(1:3);
h = x(4:6);
n = x(7:9);
mP= x(10);

% System Parameters
[T,~,md,~,~,~,~]=params(n);

% Rocket Masses
[mF,mS,mT]=rocketmass(r,h,n);
```

```
% Burn Times
tb = mF./md;

% Dry Stage n Masses
mE1 = mP+(mS+mT)*[1;1;1]+mF*[0;1;1];
mE2 = mP+(mS+mT)*[0;1;1]+mF*[0;0;1];
mE3 = mP+(mS+mT)*[0;0;1]+mF*[0;0;0];

mE = [mE1, mE2, mE3];

% Delta-V Calculations
dV = (1000*T.*tb).*log((mE+mF)./mE)./mF;

% Thrust to Weight Ratio Calculations
TWR= (1000/9.81)*(T./(mE+mF));

% Acceleration Calculations
Acc= (1000/9.81)*(T./mE);

% Nonlinear equality constraints and their gradients
c1 = sum(dV)-9000;
ceq= [c1];

% Nonlinear inequality constraints and their gradients
c1 = sum(tb)-800;
c2 = 1-TWR(1);
c3 = 1-TWR(2);
c4 = 1-TWR(3);
c5 = Acc(1)-6;
c6 = Acc(2)-6;
c7 = Acc(3)-6;
c8 = r(2)-r(1);
c9 = r(3)-r(2);
c  = [c1,c2,c3,c4,c5,c6,c7,c8,c9];

end

function [T,MT,md,rhox,rhof,VFR,RS]=params(n)
% System Parameters per number of thrusters
T = [6804, 880, 1000].* n;           % Thrust (kN)
MT= [8353, 1789, 1789].* n;          % Motor Mass (kg)
md= [2618, 248.6, 248.6].* n;        % Mass flow rate (kg/s)

% Fuel Parameters
rhox = [1141, 1141, 1141];           % Density [LOX, LOX, LOX]
(kg/m3)
rhof = [915, 70.85, 70.85];          % Density [RP1, LH2, LH2]
(kg/m3)
VFR  = [1.82, 0.34, 0.34];           % Fuel Ratio [LOX/RP1, LOX/LH2]

% Structural Parameters
RS   = [72.21, 36.57, 70.48];        % Mass/unit Surface Area
(kg/m2)
```

```
end

function [mF,mS,mT]=rocketmass(r,h,n)
% Thruster Mass and other Parameters
[~,mT,~,rhox,rhof,VFR,RS]=params(n);

% Fuel Mass
mF = pi*(rhox.*VFR+rhof).*((h-3.8*r).*(r.^2)+2.667*(r.^3))./(VFR+1);

% Structural Mass
mS = 2*pi*RS.*(h.*r+r.^2);

end
```

A3. MATLAB CODE – OLB PARAMETRIC STUDY

```
close all

Data = csvread('MatData.csv');

M_pay = Data(:,1);

M_OLB = Data(:,2);
h_OLB = Data(:,3);

D_max = Data(:,4);
D_min = Data(:,5);

t_BRN = Data(:,6);
n_S01 = Data(:,7);
n_S23 = Data(:,8);

%% Plot the Results

figure
subplot(2,1,1)
plot(M_pay,M_OLB,'linewidth',2);
title('Total OLB Mass')
xlabel('Payload Mass (metric tons)')
ylabel('OLB Mass (metric tons)')
axis([0 120 0 4000])
grid on

subplot(2,1,2)
[Ax,L1,L2] = plotyy([M_pay,M_pay],[D_max,D_min],M_pay,h_OLB);
title('General OLB Dimensions')
xlabel('Payload Mass (metric tons)')
ylabel(Ax(1),'OLB Diameter (m)')
ylabel(Ax(2),'OLB Stage Height (m)')
axis(Ax(1),[0 120 0 12])
axis(Ax(2),[0 120 40 80])
set(L1,'linewidth',2)
set(L2,'linewidth',2)
legend('Maximum Diameter','Minimum Diameter','Overall
Height','location','northwest')
grid on

figure
subplot(2,1,1)
plot(M_pay,t_BRN,'linewidth',2);
title('Booster Burn Time')
xlabel('Payload Mass (metric tons)')
ylabel('Burn Time (s)')
axis([0 120 0 800])
grid on

subplot(2,1,2)
[Ax,L1,L2] = plotyy(M_pay,n_S01,M_pay,n_S23);
```

```
title('General OLB Dimensions')
xlabel('Payload Mass (metric tons)')
ylabel(Ax(1),'Stage 1 Thrusters')
ylabel(Ax(2),'Combined Stage 2 and Stage 3 Thrusters')
axis(Ax(1),[0 120 0 8])
axis(Ax(2),[0 120 4 10])
set(L1,'linewidth',2)
set(L2,'linewidth',2)
grid on
```

A4. MATLAB CODE – PLS SOLVER

```
function [X,fval]=MARS_PSLv3()
%%Maximum number of iterations that the solver will run
iterations = 10;

%%Filename to load
file = 'cargo_simplified';

%%Maximum Parameters of Shuttle
Ms = 9770;
Vs = 79.87;

%%Cargo Breakdown
cargo = load(file);
cargo = cell2mat(struct2cell(cargo));
mass_cargo = cargo(:,1);
vol_cargo = cargo(:,2);
% MTBF = cargo(:,3);
MTBF = [5.42, 3.76, 3.76, 3.12, 2.92, 5.71, 7.61]';

%%Mission Info
tc  = 1; %ceil(26*30.416666667);     %Mars One launch cycles/ Time per
cycle (days)
t_end = 22; %7300;                    %number of cycles in 22 year plan
M = (length(cargo));                %number of devices
T = length([1:t_end-tc+1]);         %number of possible trips

X=ones(T*M,1);                        %variable matrix

%%Mass Restriction (A*X-b)
mass = repmat(mass_cargo,T,T)';
A=kron(eye(T),ones(1,M)).*mass;     %mass for each variable
b = repmat(Ms,T,1);                 %mass restriction

%%Volume Restriction (C*X-d)
vol = repmat(vol_cargo,T,T)';
C=kron(eye(T),ones(1,M)).*vol;      %vol for each variable
d = repmat(Vs,T,1);                 %vol restriction

%%MTBF Restriction (E*X-f)
X0 = ones(M,1);
MTBF0 = (X0.*MTBF);
G0 = repmat(MTBF0,t_end-tc+1,1);          %timeframe with no arriving
supplies
g0a = [tc:t_end];                   %duration of timeframe with no
supplies
g0b = repmat(g0a,M,1);        %each variable
g0 = reshape(g0b, M*(t_end-tc+1),1);        %reshapes into array
f = G0-g0;                          %device lifeframe before first supply
run

T1=t_end-tc+1;
```

```
block1 = eye(M);
block2 = tril(ones(T1,T1),-1)+eye(T1,T1);
ea = kron(block2,block1);
eb = diag(MTBF);
ec = repmat(eb, T1, T1);
E = ea.*ec;

% Linear equality constraints of the form Ax = b
Aeq = [];
beq = [];

% Linear inequality constraints of the form Ax <= b
Ain = [A;C;-E];
bin = [b;d;f];

% Nonlinear constraints are defined by subfunction nonlcon

% Bounds on the design variables
lb = [0*X];
ub = [];

% Implement the solver
% % options = optimoptions('ga','MaxFunEvals',iterations); %<- NOT SURE
IF THIS WORKS
IntCon = linspace(1,length(X),length(X));
[X,fval] = ga(@fun,ceil(X),Ain,bin,Aeq,beq,lb,ub,@nonlcon,IntCon);

% options =
optimoptions('fmincon','Algorithm','sqp','MaxFunEvals',10000);

% [X,fval] =
fmincon(@(X)fun(X,T,M),X*0,Ain,bin,Aeq,beq,lb,ub,@nonlcon,options);
end

function f=fun(X,T,M)
f=sum(sum(X))+1e3*sum((kron(eye(T),ones(1,M))*X)>0);
end

function X=xval(x)
X=ceil(x);
end

function [c,ceq]=nonlcon(X)
c=[];
ceq=[];
end
```