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OPTIMIZATION OF FUEL EFFICIENCY OF A HYBRID ELECTRIC BUS

By

Govind Menon
Jonathan Verghese
Karan Shah
Shanker Krishnan

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ABSTRACT

A bus is one of the most popular and widely used modes of public transportation. It is also one of the main contributors of CO2 emissions. An estimated 4400kg/person/year of CO2 is emitted by a bus in the United States of America alone. From this it is imperative that a more eco-friendly version of the bus is the need of the hour. Although the use of hybrid electric powertrains has been comparatively more popular in the light motor vehicle category, the use of such technology is not predominantly found in urban transit buses. We aim to shorten this gap with this project.

The aim of this project is to optimize the hybrid electric system to minimize the fuel consumption. The project involves optimization process of mainly 3 subsystems. These include a) the mechanical coupler, b) the optimal control and c) the battery sizing, all of which play a crucial role in determining the mileage of the bus. The modelling of this problem is done by collecting the drive cycle data of a bus and doing the necessary modelling using MATLAB. The final mpg value obtained is 7.5 which is noticeably better than a regular transit bus which has a mpg value of 4.03(orange county drive cycle). The subsystem optimization values from the mechanical coupler and the battery subsystem are fed into the optimal control code to find the overall system optimization.

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ACKNOWLEDGEMENT

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1. DESIGN PROBLEM STATEMENT

The objective of the project is to optimize of fuel efficiency for a hybrid electric bus operating in urban driving conditions as per a specific urban drive cycle observed. The hybrid electric bus taken into consideration has a parallel architecture powered by a single internal combustion engine and single motor configuration whose outputs are coupled by a mechanical coupler.

The global objective of optimization of the fuel efficiency is further sub-divided into individual subsystems which are the battery subsystem, the mechanical coupler subsystem and optimal control subsystem and they have their respective objectives. The optimum results obtained from the individual subsystems are integrated into a common objective function which depends on the design variables and parameters from all the three subsystems to obtain the optimal solution for the fuel efficiency of the hybrid electric bus.

The objective of the Battery subsystem is to maximize the power generated by the battery. This leads to an overall increase in the electric energy and therefore less fuel is consumed. The battery however has to be operated within its ideal operating conditions. One way to increase the usage if the electric power would be to just use a big battery but this in turn leads to an increase in weight which causes an increase in fuel consumption of the bus. The battery will be completely used in low power requirement situations and partially when the power requirement increases and keeps maintaining the engine operation in an efficient region. Another consideration would be to maintain the State of Charge (SOC) of the battery within defined limits.

The Mechanical Coupler subsystem mainly focuses on minimizing the weight of the mechanical coupler (W) by optimizing for the dimensions of the spur gears used which directly depend on the torques supplied both by the motor and the engine which are incorporated in a parallel architecture of the HEV bus as per the specific drive cycle observed for current urban driving conditions for a city bus. These variables directly impact the overall weight of the vehicle. Reducing the volume and the weight of the mechanical coupler thereby impacts the fuel efficiency (MPG) of the vehicle which meets the primary objective of the project, i.e. to optimize the overall fuel efficiency of a Hybrid Electric Bus.

The optimal control subsystem aims at devising the optimal control strategy for utilizing the Motor and the Internal Combustion engine in an effective way to

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increase fuel efficiency and to meet the required T and ω characteristics as for the best fuel efficiency to be achieved there has to be proper management of the engine as well as the electric motor. The tradeoffs have to be made between SOC, battery subsystem and G, FR with the mechanical coupler subsystem. The first step in the optimizing process would be to generate a graph of torque (T) vs. the Angular velocity (ω) of the engine. This is done with the help of the engine data. With the help of this data and the data form the drive cycle T_m and ω_m is generated for different values of T. With the data that has been collected a graph between ΔP and fuel cost is plotted and the optimum points are determined. For the optimizing target to be achieved the SOC at the beginning and the end must be at 80% and should be kept between prescribed bounds at all times.

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2. NOMENCLATURE

2.1 NOMENCLATURE – A (BATTERY SUBSYSTEM)

n_s - Number of cells in series

n_p - Number of cells in parallel

P_{bat}- Power generated by the battery

P_{reg}- Power generated by regenerative braking

SOC_{min} - Minimum SOC

SOC_{max} - Maximum SOC

m_{bat} - Mass of each cell

M- Mass of the entire battery

V_{oc} - Open Circuit voltage

R - Battery internal resistance

Q - Battery Capacity

V_{bat} - Nominal Cell voltage

C_{max} - Maximum discharge rate of the cell
C_{min} - Minimum discharge rate of the cell

I_{dmax} - Maximum discharge current

I_{cmin} - Minimum charging current

w - Weight factor

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2.1 NOMENCLATURE – B (MECHANICAL COUPLER SUBSYSTEM)

R = Pitch Radius of the output gear of the mechanical coupler, mm
r = Pitch Radii of the input gear coupled with the output shafts of the engine and motor respectively, mm
W = Weight of the mechanical coupler, Kg
m = Module of the gears used, mm
b = Width of the gear teeth, mm
G = Gear ratio between the input gears and the output gear of the mechanical coupler, dimensionless
ρ = Density of the material used for the gears and pinion, Kg/mm^3
σ_{bi} = Bending stress induced on the input gear teeth, N/mm^2
σ_{bo} = Bending stress induced on the output gear teeth, N/mm^2
σ_{TU} = Maximum allowable bending stress for the input and output gear material, N/mm^2
σ_c = Contact Surface stress induced on the input and output gears teeth, N/mm^2
σ_{endl} = Endurance limit of the input and output gear material, N/mm^2
E = Young's Modulus of the material used for the gears, N/mm^2
Φ = Pressure Angle of the gear teeth, Degrees
y = Lewis form factor for gear teeth, Dimensionless
g = Acceleration due to gravity, mm/s^2
FR = Final drive ratio of the vehicle, dimensionless
R_w = Radius of the wheels, mm

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2.3 NOMENCLATURE – C (OPTIMAL CONTROL SUBSYSTEM)

W = Kerb weight of the bus, Kg
T_d =torque demand at the wheels , Nm
T_e =torque provided by the engine , Nm
ω_d=angular velocity demand , rad/s
ω_d=Angular velocity of the engine , rad/s
FR =Final drive ratio
N_s =Number of cells in series
N_p =Number of cells in parallel
P =power of battery , kW

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3. BATTERY SUBSYSTEM – JONATHAN VERGHESE

3.1 Mathematical Model

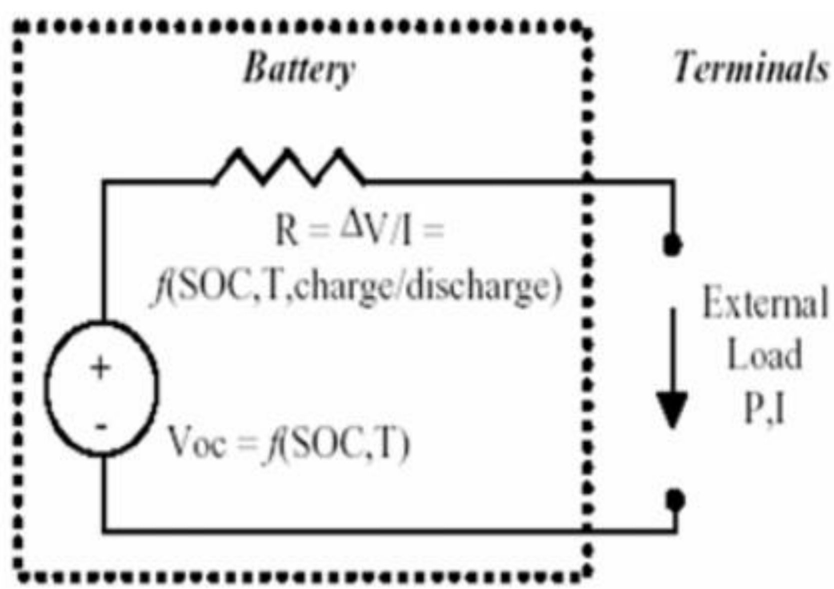
Objective Function

The objective would be to utilize the battery as much as possible. So this results in lesser fuel consumption. The power requirement can be determined by the drive cycle.

$$min\, f = P - (Pbat + Preg)$$

Where P is the power requirement of the vehicle. Since this is always a number we can neglect it from the objective function. Also another point to note is that the only time that the battery is recharged during operation is during regenerative braking.

$$min\, f = -(Pbat + Preg)$$



The battery is modeled as a static circuit. The internal resistance is depicted as R in the figure. The open circuit voltage is represented as $V_{OC.}$ Both of these parameters are functions of the SOC. They are also both functions of temperature but for this study we neglect the effects of temperature and assume it as a constant. We also assume the resistance of the battery during charging as well as discharging are the same.

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We can write the power generated in a single cell with the formula.

$$P = (V_{oc} I_{bat} - I_{bat}^2 R)$$

When we solve for the roots of this equation we get

$$I_{bat} = \frac{V_{oc} - \sqrt{V_{oc}^2 - 4RP_{bat}}}{2R}$$

$$V_{bat} = I_{bat} R = \frac{V_{oc} - \sqrt{V_{oc}^2 - 4RP_{bat}}}{2}$$

Discharge rate of the battery is given by

$$SOC = \frac{I_{bat}}{C_{max}}$$

Also as previously mentioned

- $V_{oc} = f(SOC)$
- $R_{bat} = f(SOC)$

Both are functions of SOC. These parameters can be found out using curve fitting to experimental data for a 9v Prius NiMh battery cell. This consists of cells in parallel, n_p , which are together called a module and many modules are connected in series. Cells in parallel determine the current requirement of the pack and cells in series determine the voltage requirement.

Adapting the results of curve fitting from a previous project.

$$V_{oc} = n_s 2(3.6061 + 0.7737SOC - 0.9123SOC^2 + 0.6771SOC^3)$$

$$R = 0.01 \frac{n_s}{n_p}(0.0486 + 1.6750SOC - 5.190SOC^2 + 4.2533SOC^3)$$

Assumptions

- The $V_{oc} and\ R$ are considered as functions of SOC and the temperature effect are not taken into consideration.
- The charging and discharging resistances are assumed to be equal.
- During operation the only way the battery is charged is by regenerative braking. This is because of the architecture of the hybrid powertrain.

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Constraints

G1: $P_{bat} - n_s V_{bat} n_p C_{max} Q \leq 0$. Power of the battery should be less than the power of the pack.

G2: $\frac{V_{oc} - \sqrt{V_{oc}^2 - 4RP_{bat}}}{2} - n_s V_{bat} \leq 0$. Voltage of the battery should be less than the voltage of the pack.

G3: $\frac{V_{oc} - \sqrt{V_{oc}^2 - 4RP_{bat}}}{2R} - n_p I_{dmax} \leq 0$. The discharging current of the battery must be less than the maximum discharge current.

G4: $n_p I_{cmin} - \frac{V_{oc} - \sqrt{V_{oc}^2 - 4RP_{reg}}}{2R} \leq 0$. The charging current of the battery must be more than the minimum discharge current.

G5: $\frac{V_{oc} - \sqrt{V_{oc}^2 - 4RP_{bat}}}{2RQn_s} - C_{max} \leq 0$. The discharging rate of the battery must be less than the maximum discharge rate.

G6: $C_{min} - \frac{V_{oc} - \sqrt{V_{oc}^2 - 4RP_{reg}}}{2RQn_s} \leq 0$. The charging rate of the battery must be more than the minimum discharge rate.

G7: $n_s n_p m_{bat} - M \leq 0$. The mass of the whole pack must be lesser than the mass limit set for the battery pack.

G8: $wrc - P_{reg} \leq 0$

H1: $V_{oc} - n_s 2(3.6061 + 0.7737SOC - 0.9123SOC^2 + 0.6771SOC^3) = 0$

H2: $R - 0.01\frac{n_s}{n_p}(0.0486 + 1.6750SOC - 5.190SOC^2 + 4.2533SOC^3) = 0$

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Design Variables

n_s - Number of cells in series

n_p - Number of cells in parallel

P_{bat}- Power generated by the battery

P_{reg}- Power generated by regenerative braking

Design Parameters

SOC_{min} - Minimum SOC

SOC_{max} - Maximum SOC

m_{bat} - Mass of each cell

M- Mass of the entire battery

V_{oc} - Open Circuit voltage

R - Battery internal resistance

Q - Battery Capacity

V_{bat} - Nominal Cell voltage

C_{max} - Maximum discharge rate of the cell
C_{min} - Minimum discharge rate of the cell

I_{dmax} - Maximum discharge current

I_{cmin} - Minimum charging current

w - Weight factor

The initial mathematical model prepared in the project proposal and the progress report contained a few more constraints that were removed to simplify the model. In the project proposal it was planned to conduct an optimization study in two different fronts i.e. the pack level design and the cell level design. The cell level design was to consider factors like anode and cathode thickness, its porosity and number of layers. The objective function that the problem was designed to minimize the volume of the cell. At the pack level design the objective was to minimize the weight by varying the module sizes depending upon requirements of the driving cycle. This problem was changed to the current status as it had more relevance to the other

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subsystems and the overall optimization of the system. The changing of the objective to maximize the power generated by the

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3.2 Model Analysis

Function Dependency Table

These functional dependency tables shows the relationship between the variables and parameters with respect to the objective function and the constraints.

1. With Variables

	n_s	n_p	P_{bat}	P_{reg}
f			*	*
G1	*	*	*	
G2	*		*	
G3		*	*	
G4		*		*
G5	*		*	
G6	*	*		*
G7	*	*		
G8				*

2. With Parameters

	M	m_{bat}	V_{oc}	R	Q	V_{bat}	C_{max}	C_{min}	I_{dmax}	I_{cmin}
f										
G1					*	*	*			
G2			*	*		*				
G3			*	*					*	
G4			*	*						*
G5			*	*	*		*			
G6			*	*	*			*		
G7	*	*								
G8										

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Monotonicity Analysis

	n_s	n_p	P_{bat}	P_{reg}
f			$(-)$	$(-)$
G1	$(-)$	$(-)$	$(+)$	
G2	$(-)$			
G3		$(-)$		
G4				
G5	$(-)$			
G6	$(+)$			
G7	$(+)$	$(+)$		
G8				$(-)$

From the monotonicity analysis it seems that the problem seems well bounded. According to MP1 every increasing variable must bounded below by a non-increasing active constraint and every decreasing variable must be bounded above by an increasing active constraint. In this case G1 is active for P_{bat}. G8 will be active with respect to P_{reg}. According to MP2 nonobjective variables should be bounded from above and below by semi-active constraints and in this case n_s and n_p are the nonobjective variables and they are bounded below and above. These results could be validated with the help fmincon solver to observe the activity of the constraints.

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3.3 Optimization Study

The fmincon solver is used for the optimization study. The interior point algorithm is used initially.

The initial guess is

x0 = [90000 -10000 80 15]; %Initial Guess

Iter	F-count	f(x)	Feasibility	First-order optimality	Norm of step
31	305	-4.199940e+04	0.000e+00	1.000e+05	1.697e-02
32	316	-4.199940e+04	0.000e+00	1.000e+05	1.755e-02
33	327	-4.199940e+04	0.000e+00	1.000e+05	1.803e-02
34	338	-4.199940e+04	0.000e+00	1.000e+05	1.837e-02
35	349	-4.199940e+04	0.000e+00	1.000e+05	1.856e-02
36	362	-4.199940e+04	0.000e+00	1.000e+05	1.857e-02
37	375	-4.199940e+04	0.000e+00	1.000e+05	1.837e-02
38	388	-4.199940e+04	0.000e+00	1.000e+05	1.797e-02
39	401	-4.199940e+04	0.000e+00	1.000e+05	1.737e-02
40	407	-4.199940e+04	0.000e+00	1.000e+05	1.413e-01
41	413	-4.199980e+04	0.000e+00	1.000e+05	1.267e-02
42	418	-4.237641e+04	0.000e+00	1.000e+05	4.238e-03
43	424	-6.228303e+04	0.000e+00	6.269e+02	2.231e-01
44	430	-6.398349e+04	0.000e+00	5.792e+01	3.153e-02
45	435	-6.399990e+04	0.000e+00	2.791e-01	2.049e-04
46	440	-6.400000e+04	0.000e+00	2.134e-03	2.745e-05

Figure 1 Iterations from 31 to 46

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in feasible directions, to within the default value of the function tolerance, and constraints are satisfied to within the default value of the constraint tolerance.

The values of first order optimality and norm of the step as governed by the KKT conditions shows that they have become small enough to make the algorithm converge.

	P_{bat}	P_{reg}	n_s	n_p
Xoptimal Values	71999.99952 2	- 8000.000018347	75.09915043	26.63145964

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These are the optimal values obtained from fmincon. The power generated calculated comes up to 72kW.

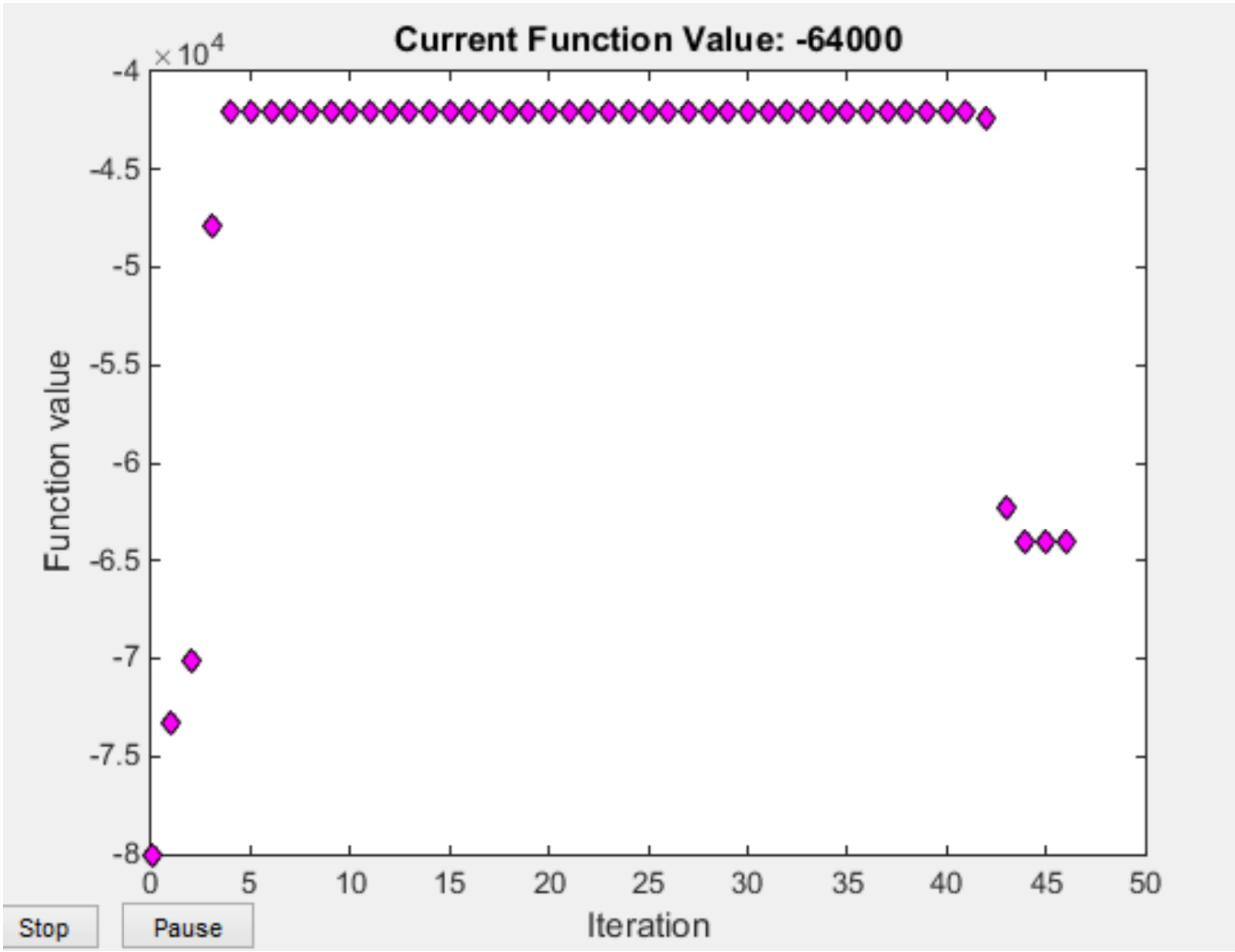

Figure 2 Plot for Function value with the iterations using the interior point algorithm

Lagrange Multipliers	At Lower Bound	At Upper Bound
n_s	1.36770609803501e-05	0.000110438217930799
n_p	7.19755819592180e-06	5.75971242691069e-06
P_{bat}	6.11699375500355e-06	0.000467108191695625
P_{reg}	3.82739646333024e-06	0.00145627272933446

This table shows the Lagrange multipliers at the upper and lower bounds. All of them are zero that means that none of the constraints are active.

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Below are the values of the hessian at the optimal solution

0.000410	0.002307	0.053238	0.062735
0.002307	0.497961	0.019243	0.026689
0.053238	0.019243	75.50823	123.8445
0.062735	0.026689	123.8445	204.7913

As it can be seen, the hessian is positive definite that shows convexity of the objective function in the constrained space.

Change in Design variables with change in the initial guess

	Starting Point 1	Starting Point 2	Starting Point 3
n_s	75	67	73
n_p	27	30	28
P_{bat}	72kW	72kW	72kW
P_{reg}	8kW	8kW	8kW

This table shows how the design variables change with the change in starting point. The values of the power from the battery and power obtained during regeneration always converge to the same values but the values of number of cells in series and parallel change.

This optimization problem was run using a few other algorithms to check validity of results obtained and to provide a comparison between the different algorithms to see the efficiency of each of them.

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Algorithm	Iterations
Interior point	46
Active Set	7
SQP	3

From the results it can be seen that the interior point algorithm takes much longer to converge in comparison to SQP and active set. This is an important consideration when the design problem is more complex than the one currently chosen. It might be more computationally expensive to use the interior point to solve this problem.

3.4 Parametric Study

This section will deal with how different sets of important parameter values affect the optimal solution of the problem and whether there is any pattern or link between the parameters of the design problem. Another study is also done by changing the value of the bounds and how this affects the optimal solution.

Mass of each cell

	50g	75g	100g
n_s	100	75	30
n_p	30	27	50
P_{bat}	108kW	72kW	54kW
P_{reg}	8kW	8kW	8kW

The column highlighted represents the values of the parameter considered for the study.

- When the mass of the cell is reduced to 50g the power of the battery pack increases as the pack can incorporate more number of cells. Looking at the values of ns, it can be seen that the value is hitting the upper bounds.
- When the mass of the cell is increased to 100g the power of the battery back reduces due to lesser number of cells. For this case the value of np hits the upper bound.

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In one case the voltage of the battery is constrained and in another case the capacity is constrained, as the number of cells in series affects the voltage of the battery and the number of cells in parallel affect the battery capacity.

Mass of the battery pack

	100kg	150kg	250kg
n_s	85*	75	100
n_p	13*	27	30
P_{bat}	50kW*	72kW	110kW
P_{reg}	8.1kW*	8kW	8kW

The column highlighted represents the values of the parameter considered for the study.

- For the case when the mass of the battery pack is 100kg the solution does not converge.
- For the case when the mass of the battery pack is 250kg the ns value hits the upper bounds because more number of cells can be added and so the voltage increases and therefore the power produced by the pack increases.

Voltage of each cell

	6V	7.2V	9V
n_s	67	75	97
n_p	30	27	21
P_{bat}	60kW	72kW	83kW
P_{reg}	8kW	8kW	8kW

The column highlighted represents the values of the parameter considered for the study.

- Understandably for the case when the voltage is 6V the power of the pack has reduced.

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- For the case when the cell voltage is 9V a higher value of power is seen. The number of cells have also increased leading to an increase in the max operating voltage.

Capacity of each cell

	3.3A-h	5A-h	10A-h
n_s	67*	75	100
n_p	30*	27	12
P_{bat}	50kW*	72kW	50kW
P_{reg}	8kW*	8kW	8kW

The column highlighted represents the values of the parameter considered for the study.

- For the case when the capacity of the cell is 3.3A-h the solution does not converge.
- When the capacity of the cell is 10A-h the ns value hits the upper bound.

The interesting observation in this parametric study is that unlike the other studies power of the battery pack does not vary linearly with change in capacity.

Varying the bounds of the P_{bat}

	Trial 1	Trial 2	Trial 3
n_s	75	85	87*
n_p	27	24	23*
P_{bat}	72kW	72kW	75kW*
P_{reg}	8kW	8kW	8kW*

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The column highlighted represents the values of the bounds considered for the study.

Trial 1: lb = [50000 -12000 50 10]; %Lower bound of decision variables
ub = [150000 -3000 100 30]; %Upper bound of decision variables
Trial 2: lb = [25000 -12000 50 10]; %Lower bound of decision variables
ub = [250000 -3000 100 30]; %Upper bound of decision variables
Trial 3: lb = [75000 -12000 50 10]; %Lower bound of decision variables
ub = [100000 -3000 100 30]; %Upper bound of decision variables

In all the cases the P_{bat} remains almost the same but the values of n_s and n_p change therefore affecting the values of max operating voltage and battery capacity.

Varying the bounds of n_s and n_p

	Trial 1	Trial 2	Trial 3
n_s	75	65	80
n_p	27	31	25
P_{bat}	72kW	72kW	72kW
P_{reg}	8kW	8kW	8kW

The column highlighted represents the values of the bounds considered for the study.

Trial 1: lb = [50000 -12000 50 10]; %Lower bound of decision variables
ub = [150000 -3000 100 30]; %Upper bound of decision variables
Trial 2: lb = [50000 -12000 50 10]; %Lower bound of decision variables
ub = [150000 -3000 150 50]; %Upper bound of decision variables
Trial 3: lb = [50000 -12000 50 10]; %Lower bound of decision variables
ub = [150000 -3000 80 25]; %Upper bound of decision variables

In all the cases the P_{bat} remains the same but the values of n_s and n_p change therefore affecting the values of max operating voltage and battery capacity. What's interesting is even after relaxing the bounds on n_s the value *decreases* and not increases but the value of n_p on the other hand *increases*. The last case as expected when the bounds were made tighter for n_s *and* n_p the values hit the upper bounds.

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3.5 Discussion of Results

- The value of the objective function from the model agrees with the practical values. The power of approximately 70 KW is required by the vehicle going at around 12 - 18 m/s.
- The number of cells in series comes out to be around 70 which leads to a max operating voltage of 540V.
- The number of cells in parallel is about 30 which leads to a max capacity of around 150Ah.
- The overall weight of the battery pack works out to 150kg.

These are the observations from the optimal solution. The results make sense. The parametric studies show that the mass hugely affects the design of the battery pack. What is also interesting is that as we increase the mass of the battery we must also increase the voltage of each cell otherwise the solution keeps hitting the bounds. Changing the voltage requires a change in design of the individual cells which is not something that can be readily changed. Also another key point to note is that for a specific cell voltage there is a sweet spot in the maximum allowable mass such that the solution does not always lie on the boundary. So there can be two approaches when designing the battery pack. Either we set the cell voltage as constant and then find the optimal mass of the battery pack. The other approach could be setting the maximum weight as a parameter and then find the optimal cell voltage.

Other things that can be looked for a future more in depth study could be study of the battery parameters at a cell level and then how this affects the battery pack design.

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4. MECHANICAL COUPLER SUBSYSTEM – KARAN SHAH

4.1 MATHEMATICAL MODEL

OBJECTIVE FUNCTION

To minimize the weight of the mechanical coupler that transmits the maximum power and the maximum torque to meet the demands of the Hybrid Electric Bus which directly affects the fuel economy of the Hybrid Electric Bus based on the urban driving conditions for a city bus.

Minimize W = $\boldsymbol{\pi * b * r^2 * \rho * (G^2 + 2)}$
W.r.t b, r, G

Where,
W = Weight of the mechanical coupler, Kg
r = Pitch Radius of the input gear of the mechanical coupler, mm
b = Width of the gear teeth, mm
ρ = Density of the material used for the input and output gears, Kg/mm^3
G = Gear ratio between the input gears and the output gear of the mechanical coupler, dimensionless

Since, the weight of the mechanical coupler directly impacts the weight of the vehicle which effectively affects the torque and speed demands of the vehicle. The torques produced by the motor and the engine are coupled by the mechanical coupler which results in a multiplied torque output due to the gear ratio of the mechanical coupler. The fuel rate consumption can be determined by the weight of the vehicle as per the speed and torque demands. Thus, the fuel consumption can be calculated for a given weight of the mechanical coupler and the optimized fuel efficiency can be obtained from the minimized weight of the coupler.

CONSTRAINTS

1. The bending stress induced on the teeth of the input gear during meshing should be lesser than or equal to the maximum allowable bending stress of the material of the gears.

$\boldsymbol{\sigma_{bi} \leq \sigma_{bmax}}$

i.e. $\boldsymbol{\sigma_{bmax} - \sigma_{bi} \leq 0}$

$$\boldsymbol{T - \sigma_{TU} * b * (0.154 * r - 0.456) \leq 0}$$

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2. The bending stress induced on the teeth of the output gear during meshing should be lesser than or equal to the maximum allowable bending stress of the material of the gears.

$\sigma_{bo} \leq \sigma_{bmax}$

i.e. $\sigma_{bmax} - \sigma_{bo} \leq 0$

$$T * G - \sigma_{TU} * b * (0.154 * r * G - 0.456) \leq 0$$

3. The Contact Surface stress induced on the teeth of the input and output gear during meshing should be lesser than or equal to the maximum endurance limit of the material of the gears.

$\sigma_{cgt} \leq \sigma_{endl}$

$\sigma_{cgt} - \sigma_{endl} \leq 0$

$$\sqrt{\frac{E * \left(T * \sqrt{r} - \sqrt{a}\right) * (G+1)}{2 * (1 - \mu^2) * \cos\emptyset * b * \sqrt{\pi^3} * \sqrt{r^5} * G * \sqrt{(1 + G^2)}}} - \sigma_{endl} \leq 0$$

4. The radii of the input gears of the mechanical coupler cannot be lesser than or equal to zero.

r > 0

-r < 0

5. The Width of the teeth of the input gears and the output gear of the mechanical coupler cannot be lesser than or equal to zero.

b > 0

-b < 0

6. The Gear ratio between the input gears and the output gear of the mechanical coupler cannot be lesser than or equal to zero.

G > 1

G – 1 > 0

-G + 1 < 0

7. The design constraint, which is the ratio of the Width of the gear to its radius is equal to the ratio of gear ratio to the sum of gear ratio and 1.

b/r = G/G + 1

b*(G + 1) – r*(G) = 0

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DESIGN VARIABLES & PARAMETERS

Design Variables:
Radii of the input gears, r
Width of the gear, b
Gear Ratio between the gear train, G
Module of the gear, m (Usually, the value of m = 1)

Parameters:
Input Torque to the input gears, T
Maximum bending stress induced on the gear teeth, σ_{bi}, σ_{bo}
Maximum contact surface stress induced on the gear teeth, σ_{cgt}
Acceleration of the hybrid electric bus, a
Pressure angle, Φ
Module of the spur gears, m
Young's Modulus of the gear material, E
Poisson's Ratio of the gear material, μ

SUMMARY MODEL

Minimize the f(x) = Weight of the Mechanical Coupler, (W) of the Mechanical Coupler, of a Hybrid Electric Bus under urban city driving conditions.

Minimize f(x) = $\pi * \rho * b * r^2 * (G^2 + 2)$
W.r.t. r, b, G
Subject to
G1 → $T - \sigma_{TU} * b * (0.154 * r - 0.456) \leq 0$
G2 → $T * G - \sigma_{TU} * b * (0.154 * r * G - 0.456) \leq 0$
G3 → $\sqrt{\frac{E*(T*\sqrt{r} - \sqrt{a})*(G+1)}{2*(1-\mu^2)*\cos\emptyset * b * \sqrt{\pi^3} * \sqrt{r^5} * G * \sqrt{(1+G^2)}}} - \sigma_{endl} \leq 0$
G4 → -r < 0
G5 → -b < 0
G6 → -G + 1 < 0
G7 → b*(G + 1) – r*(G) = 0

`

EVOLUTION OF THE PROBLEM AND LIST OF ASSUMPTIONS

1. The optimization problem is formulated for minimization of weight of the mechanical coupler transmission system for the hybrid electric bus. The constraints are directly dependent on the input torque provided to the coupler. Initially, the torque was taken as the maximum value demanded by the vehicle. Over the span of the project, it was realized that the torque values varies and is dependent on the acceleration of the vehicle. This was the major evolution that occurred in the problem formulation of the optimization project.
2. The values for ρ, σ_{bmax} & σ_{cmax} will be obtained from standard mechanical and material properties for the chosen material for the design of the gear train of the mechanical coupler.
3. The values for m, Φ will be determined based on the type and size of the final gears selected for the gear train of the mechanical coupler.
4. The geometry of the Spur Gears design is considered to be cylindrical for ease of weight and volume calculations.
5. The gears are involute and cycloidal in geometry.
6. The pressure angle (Φ) of the gears is taken as 20°.
7. Module of the spur gears (m) is taken as 1 mm.
8. The thickness of the gear is assumed to be equal to the face width of the gear teeth.
9. The thickness or the width (b) of the gears is taken as equal for both input and output gears.
10. The material used for the gears is 4130 Chrome Moly Steel.
    1. Material Properties of 4130 Chrome Moly Steel are as follows:
    2. Ultimate Tensile Strength ($\boldsymbol{\sigma_{TU}}$) = 620 N/mm^2
    3. Endurance Limit ($\boldsymbol{\sigma_{endl}}$) = 480 N/mm^2
    4. Young's Modulus (**E**) = 200000 N/mm^2
    5. Density (**ρ**) = 0.78×10^{-5} Kg/mm^3
11. Both the input gears are identical in dimensions.
12. Both the input gears are running at identical RPM when operated together.
13. Friction losses, heat losses, energy losses and the efficiency of the transmission are neglected during the optimization process.

`

4.2 MODEL ANALYSIS

The mathematical model formulated to carry out the optimization of the mechanical coupler of the Hybrid Electric Bus yields the following results on the basis of monotonicity analysis.

- The objective function **f(x) =** $\boldsymbol{\pi * b * g * R^2 * \rho * (G^2 + 2)}$ is monotonic with respect to the design variables R, b, G.
- The monotonicity table (refer to the appendix to view the monotonicity analysis) obtained by applying the monotonicity principles to the objective functions f(x) and also to the corresponding constraints of this optimization mathematical model.
- The monotonicity analysis of the mathematical model of the optimization problem also shows that the problem is well constrained subjected to a number of constraints.
- The design variable r shows that it is an increasing function and is constrained from the bottom. Therefore, the variable moves upwards in the positive direction to obtain its optimal value.
- The design variable b shows that it is an increasing function and is constrained from the bottom. Therefore, the variable moves upwards in the positive direction to obtain its optimal value.
- The design variable G however, shows that it is a decreasing function and is constrained from the above therefore, the variable moves downwards in the negative direction to obtain its optimal value.

The objective functions and the constraints are bounded optimization problems and since they are linear equations of first order, they are monotonic in nature with respect to the design variables. The various assumptions as discussed in the previous section of the report like, the shape, geometry and the profile of the gears, the material properties, the maximum torque, the pressure angle were substituted into the various formulae to simplify the constraint equations in order to carry out the iterations of the optimization problem cost efficiently.
All the dimensions of the various parameters and variables used in the optimization problem were manually scaled into a uniform system of units and therefore, no scaling functions were used to scale the objective function and the constraints.

The variables and parameters are all converted and scaled into the following set of uniform dimension system based on the category they belong. Length,

`

mm; Weight, Kg; Stresses/Strengths, N/mm^2; Density, Kg/mm^3; Acceleration, mm/s^2

4.3 OPTIMIZATION STUDY

The following section of the report focusses on the optimization method used to minimize the weight of the mechanical coupler and the results obtained from the optimization problem.

The optimization problem has one objective function with three inequality constraints and one equality constraint like explained earlier in this report. This indicates that this is a constrained optimization problem with non-linear constraints. Therefore, the optimization is carried out by the Optimization toolbox in MATLAB using the 'FMINCON – Constrained nonlinear minimization' solver.

The 'FMINCON' solver uses 4 types of algorithms to carry out the optimization process which are namely, Interior Point, SQP, Active Set, Trust Region Reflective algorithms. The results obtained using these four algorithms will be compared to find the optimized solution for the design variables which will in turn yield the optimal weight of the mechanical coupler of the hybrid electric vehicle.

The optimization is carried out for three different starting points. Refer to the appendix of this report to view the various plots and results for the design variables and the weight of the coupler. The final results and a further analysis of the results obtained are explained precisely below.

Radii of the input gears, r = 75 mm

Radius of the output gear, R = 150 mm

Width of the input gears, b = 45 mm

Gear Ratio of the mechanical coupler = 1.6

Weight of the mechanical coupler = 27.46 Kg

Command Window Output from MATLAB:
Diagnostic Information (Output using Optimization Toolbox)

Number of variables: 3

Functions

Objective: PROJFUN (See appendix for the objective function code)

Gradient: finite-differencing

Hessian: finite-differencing (or Quasi-Newton)

`

Nonlinear constraints: PROJNONLCON (See appendix for the Nonlinear constraints code)
Nonlinear constraints gradient: finite-differencing

Constraints
Number of nonlinear inequality constraints: 3
Number of nonlinear equality constraints: 1
Number of linear inequality constraints: 0
Number of linear equality constraints: 0
Number of lower bound constraints: 0
Number of upper bound constraints: 0

Algorithm selected
Sequential quadratic programming

End diagnostic information

Optimization completed: The relative first-order optimality measure, 6.290114e-08,
is less than options.TolFun = 1.000000e-06, and the relative maximum constraint violation, 2.000687e-14, is less than options.TolCon = 1.000000e-06.

Optimization Metric		Options
relative first-order optimality =	6.29e-08	TolFun = 1e-06 (default)
relative max(constraint violation) =	2.00e-14	TolCon = 1e-06 (default)

OUTPUT USING THE 'FMINCON' SOLVER CODE

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in feasible directions, to within the default value of the function tolerance, and constraints are satisfied to within the default value of the constraint tolerance.

<stopping criteria details>
Active inequalities (to within options.TolCon = 1e-06):
lower upper ineqlin ineqnonlin

`

3

```
xopt =
  45.4387   74.5094    1.5630
fval =
  27.4648
exitflag =
     1
output =
         iterations: 9
          funcCount: 40
       lssteplength: 1
           stepsize: 1.0177e-04
          algorithm: 'medium-scale: SQP, Quasi-Newton, line-search'
      firstorderopt: 6.8581e-07
    constrviolation: 1.2810e-09
            message: [1x783 char]

lambda =
         lower: [3x1 double]
         upper: [3x1 double]
         eqlin: [0x1 double]
      eqnonlin: 1.7423e-07
       ineqlin: [0x1 double]
    ineqnonlin: [3x1 double]

grad =
    0.6044
    0.7372
   19.3237

hessian =
    0.0234    0.0233    0.6550
    0.0233    0.0299    0.7729
    0.6550    0.7729   29.6534
```

`

ANALYSIS OF RESULTS

The results provided above explicitly explains various aspects of the optimization process.

- The third inequality constraint is active for this optimization problem which is also justified from the monotonicity analysis of the optimization problem. Therefore, since the constraint is active, there exists a positive Lagrange multiplier as per the rules of nonlinear constrained optimization.
- The value of the Lagrange multiplier is obtained as 1.7423e-07.
- The results explains that the optimization completed as local minimum was found which satisfies the constraints. Also, The relative first-order optimality measure, 6.290114e-08, is less than options.TolFun = 1.000000e-06, and the relative maximum constraint violation, 2.000687e-14, is less than options.TolCon = 1.000000e-06. This indicates that the results are numerically stable and within the tolerance limits.
- The same results are obtained when the Active Set Algorithm is used for the optimization problem. Refer to the appendix of this report to view these results. As per MATLAB, Active Set algorithm is based on the KKT conditions. Since, the optimization completed as the local solution is found that satisfies the constraints and is also within the tolerance limits. Therefore, it can be concluded that the optimum solution is obtained that satisfies the KKT solutions.
- The optimization is carried out by taking into consideration three different starting point. For all the starting point combined with the various algorithms to solve the optimization problem, the values for the design variables converge at the same point which is the global optimal solution for the optimization problem. Refer to the appendix of this report to view all the results.
- The monotonicity results agrees with the numerical values obtained from the result obtained. Both the design variables, i.e. the radii (r) and the width (b) of the input gears are monotonically increasing variables while the gear ratio of the coupler is monotonically decreasing function.

4.4 PARAMETRIC STUDY

The optimization model's parametric study can be carried for the set of parameters like material properties and acceleration of the hybrid electric bus and the gear design parameters is explained below in the form of comparison study between two different sets pf parameter values. The optimization is carried

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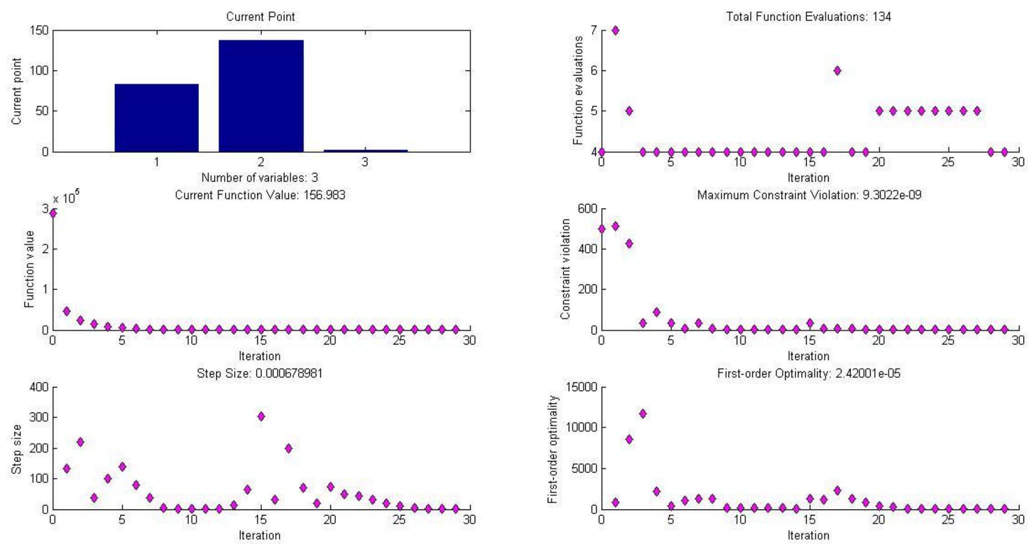
out using the SQP Algorithm which is an efficient algorithm with a starting point [1000 1000 10]
For a fixed set of parameters like:

i. **Material = Grey Cast Iron; E = 160 GPa; Ultimate tensile Strength = 430 MPa; Endurance Limit = 170 MPa, Density = 0.72 Kg/mm**

Results obtained:

b = 84 mm; r = 137 mm; G = 2.4; W = 157 Kg;

Plot:



ii. **Material = Carbon Steel; E = 190 GPa; Ultimate tensile Strength = 550 MPa; Endurance Limit = 340 MPa, Density = 0.78 Kg/mm**

Results Obtained:

b = 57; r = 91; G = 1.9; W = 51 Kg;

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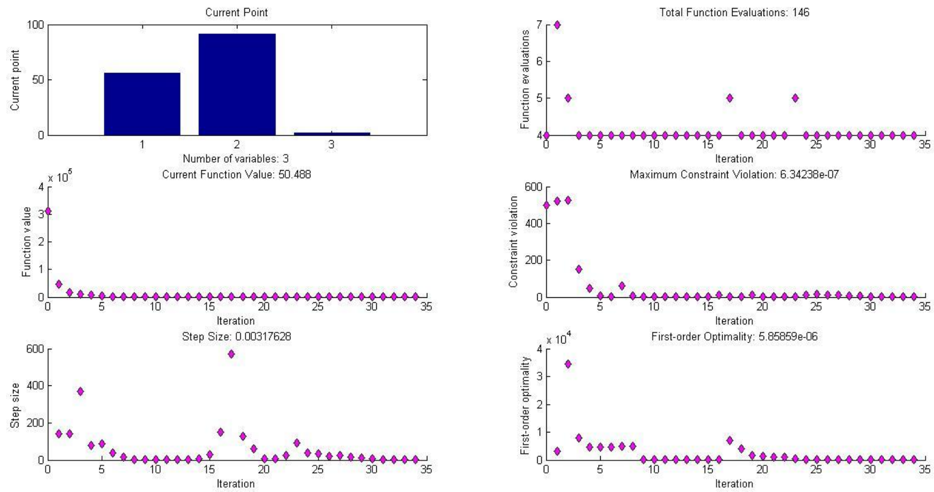
Plot:



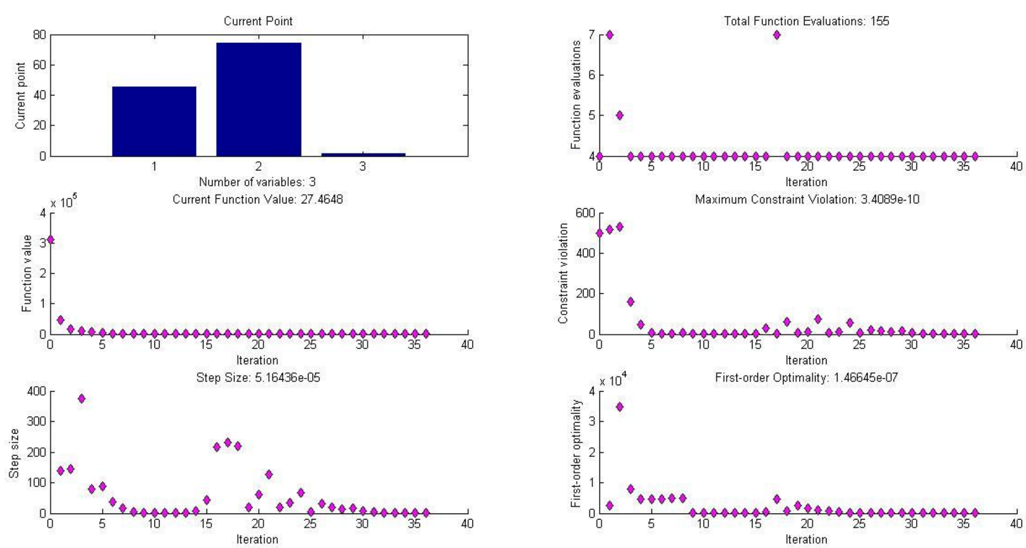
iii. Material = 4130 Chome Moly Steel; E = 206 GPa; Ultimate tensile Strength = 620 MPa;

Endurance Limit = 480 MPa, Density = 0.78 Kg/mm

Results Obtained:

b = 45 mm; r = 75 mm; G = 1.6; W = 27.46 Kg;

Plot:



The iterations while carrying out the optimization for the weight of the mechanical coupler showed the following trend. As the material properties parameters increased in numerical value for the ultimate tensile strength,

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Endurance limit and Young's Modulus, the dimensions of the design variables like radii and width of the input gear decreased however, the gear ratio of the coupler also reduced. Therefore, this shows that there is a tradeoff between the gear ratio and the dimensions of the input gears. The Weight of the coupler also reduced as the material properties increased in the numerical value.
In simple words, the weight obtained when grey cast iron was used as a material was the maximum in comparison to the weight obtained when carbon steel was used as the material. The weight further reduced when the material used was 4130 Chrome Moly steel which has been used as the gear material for this optimization project.

Therefore, the results can be generalized as the material which has the greater Ultimate Tensile Strength, Endurance Limit and Young's Modulus will have the lowest weight when used as the gear material for the mechanical coupler.

The ranges for the parameters obtained from the above results are the following:
Ultimate Tensile Strength range: 430 MPa – 1100 MPa
Endurance Limit range: 170 MPa – 550 MPa
Young's Modulus range: 160 GPa – 210 GPa
Weight Range: 160 Kg – 16 Kg
Gear Ratio Range: 2.6 – 0.7

Note: A gear ratio lesser than 1 is not acceptable as the purpose of the mechanical torque coupler is lost, as the output torque from the mechanical torque coupler will be lower than the input torque provided to the coupler.

4.5 DISCUSSION OF RESULTS

- The design implications that can be observed from the above obtained results based on constrained optimization as well as parametric study is that, the weight of the coupler is reduced as the material used for manufacturing the spur gears has greater material and mechanical properties, i.e. weight of the coupler is inversely proportional to the strength, toughness and elasticity of the material.
- The design rule identified from the solutions obtained is the for having an optimum weight for a mechanical torque coupler, the ideal gear ratio between the gear train should be in the range of 1.5 – 2.0. This implies that the output gear of the coupler should have number of teeth and its pitch radius 1.5 - 2.0

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times the number of teeth and the pitch radius of the input gear respectively. Also, another identified rule as discussed in the previous implication is that weight decreases as the material becomes stronger and tougher.

- The results do make sense as the dimensions and the weight obtained can be easily comparable to the existing gear trains that are manufactured for the commercial hybrid electric buses.
- The model limits the solutions as the constraints are to be satisfied since the optimization problem is constrained optimization. Therefore, the solution to the optimization model can be obtained only in the range of points that satisfy all the constraints and provide the minimized weight for the mechanical coupler.

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5. OPTIMAL CONTROL SUBSYSTEM – GOVIND V MENON / SHANKER KRISHNAN

5.1 MATHEMATICAL MODEL

OBJECTIVE

Optimal control subsystem controls what to use when. It determines whether the bus should be powered by the engine or the motor at any given time interval in the EPA drive cycle. The main aim of this subsystem is to integrate the other systems which the optimal control and provide an optimum values of mpg by incorporating the ECMS algorithm and Pareto curves

CONSTRAINTS

- The main constraints involved in the optimal control are the Torque limits of the motor and the engine. This is calculated by basically selecting the motor and the engine and then generating torque limit curves.
- The motor that was used was basically an I-SAM motor. The top torque limits of the motor were determined from a speed of 650 rpm to 2700 rpm. The remaining values of the torque limit was interpolated linearly to get the values of the torque limit for ''W'' ranging from 0-3300 rpm.
- The values of Ns and Np are limited by the battery constraints which also contribute to the weight (Kerb weight)

▪ $T_d = T_m + T_e$

(Total torque demand is equal to the torque of engine + torque of the motor)

▪ $T_{e\,min} <= T_e <= T_{e\,max}$

(Torque of the engine must lie between the maximum and minimum value)

▪ $T_{m\,min} <= T_m <= T_{m\,max}$

(Torque of the motor must lie between the maximum and the minimum value)

▪ $\omega_{e\,min} <= \omega_e <= \omega_{e\,max}$

(The angular velocity of the engine must lie between the maximum and the minimum value)

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- $\omega_{m\ min} <= \omega_e <= \omega_{e\ max}$

(The angular velocity of motor must lie between the maximum and the minimum value)

- ***5 < F.R < 15***

(The values of the final drive ratio is limited in the ration of 8-15)

- ***15< N_s < 50 and 10 < N_p < 30***

ASSUMPTIONS

Some of the assumptions that were made in the optimal control algorithm are as follows

- It is assumed that the bus follows the exact EPA cycle and does not deviate from it at all times
- The SOC of the battery is set to a value of 60% at the beginning and at the end
- It is assumed that the bus is operating at a constant Kerb weight.
- We assume the ideal case of the engine where we are not including the efficiency of the engine. This basically means that we are not considering the various factors like friction, fuel quality etc.
- The values of the EPA drive cycle are set for a particular urban bus in orange county and may not be the same elsewhere
- The linear interpolation made during the various segments of the program which may not be actually true in practical cases
- The motor efficiency meaning is always taken as 95% of the calculated efficiency

DESIGN VARIABLES AND PARAMETERS

The main design variables that are used are

1. ω_m (rpm)
2. ω_e (rpm)
3. T_m (Nm)
4. T_e (Nm)
5. T_d (Nm)
6. N_s, N_p

`

DESIGN PARAMETERS

1. Final drive ratio (FR)
2. Maximum Engine torque ($T_{e\ max}$) – 800 Nm
3. Maximum Engine angular velocity ($\omega_{e\ max}$) -1600 rpm
4. Maximum Motor output – 120 Kw – VOLVO I SAM electric motor
5. Motor Torque range (T_m) – 800/1200 Nm

5.2 MODEL ANALYSIS

Since the subsystem mainly deals with the programing part it does not have a mathematical formula to indicate its working pattern

- The main objective of the optimal control is to use the ECMS algorithm to determine the maximum fuel efficiency and at the same time conserving the battery use

- Optimization is done by creating grids of look up tables of the fuel efficiency and the power of the battery which are used to obtain the values of the drive mode characteristics at a certain point
- The maximum current consumed by the motor is set between two bounds to prevent infeasible mode/operating points to appear in the look up table

- The values of optimum point is developed using the equation

❖ **EC = Fuel + λ*Power of battery**

- The value of lambda is calculated using a simple root finding algorithm to determine whether the SOC reaches the required values at the end of the cycle.

5.3 BASIC FLOW OF THE PROGRAM

The optimal control subsystem is made up of various sub modules that determine the characteristics of the engine and the motor at a particular time. Based on these operating conditions the optimum mode is selected using the

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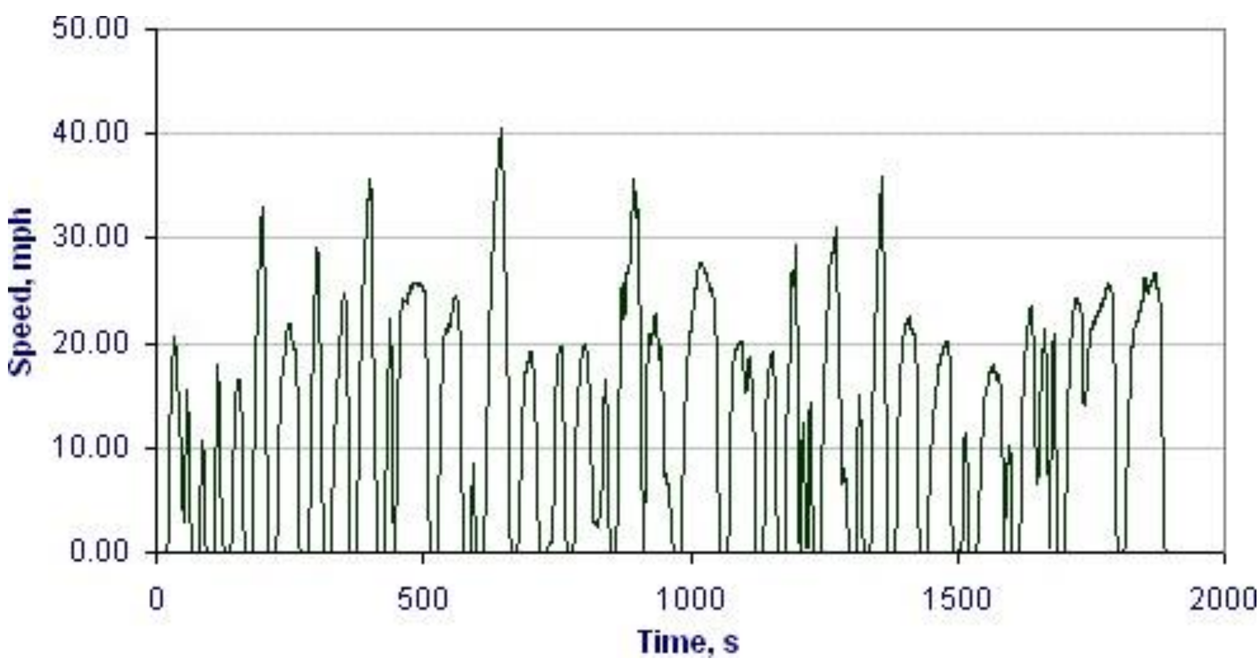
ECMS algorithm. The main sub modules that are being considered for the project are

1. Engine control
2. Motor control
3. Transmission control
4. Battery power control

Optimal control is centrally controlled by a main program which reads the values from the EPA drive cycle and computes the values of the torque and the speed required by the bus. This program requires the following inputs

- Values of the mass , drag coefficient , wheel radius , final drive ratio
- EPA drive cycle . The cycle used for the project is the Orange County transit bus drive cycle



- Torque and angular velocity of the engine
- Torque and angular velocity of the motor

On providing the inputs the program calculates the values of the Torque and the speed that is required at the wheels. This data is stored and then used to call the subprogram that calculates the Engine characteristics at that point.

Once the Engine subprogram receives the input from the main program we first estimate the fuel that is consumed by the engine for various values of torque

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and angular velocity. These values of the fuel consumption are stored in a grid which is used as a look up grid later. On calculating the engine torque we can understand the deficient torque which is required to power the bus at that point of time.

This deficiency of the torque ids satisfied by the motor. The motor program is used for this purpose. It calculates the amount of torque provided by the motor and estimates the current that is required or used up by the motor at that point. This value of current is used to determine the battery requirement and this leads us to a Battery power grid.

Thus after running these subprograms we now have a grid f fuel consumption and power of battery. These grids are used to create a Pareto curve. A Pareto curve uses a Pareto front, what this does is that it helps making tradeoffs and selecting the most feasible points among all the sets of points so that we do not have to go through the entire grid.

Along with these subprograms there is a section of coding in the optimal control that helps in the regenerative braking. This is based on the fact that every downfall of the EPA drive cycle curve denotes a point where the brake is applied or when the bus is decelerating. At these points the energy from the motion is used to recharge the battery using a simple motor that acts as a generator. This helps in improving the charge in the battery.

After we generate the Pareto curves for the entire drive cycle we perform the ECMS on these points. The ECMS algorithm helps in selecting the optimum mode that is required at each time step of the operation of the bus so that maximum fuel efficiency is attained and at the same time the battery holds enough charge. This is done based on the SOC values which are taken into consideration while calculating the Equivalent consumption

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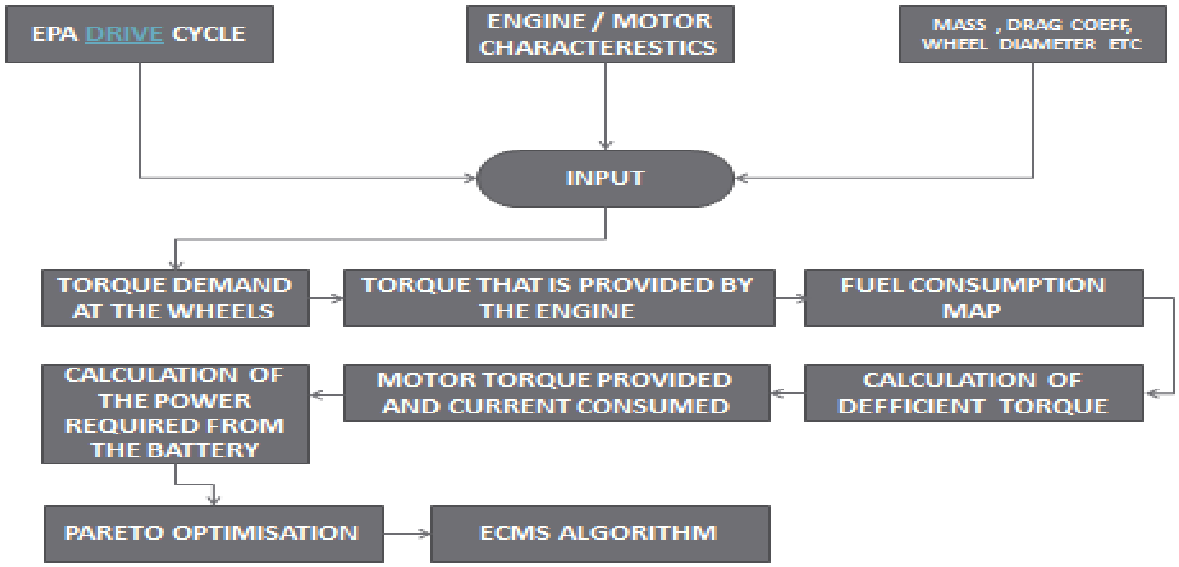
Thus the basic flow of this subsystem can be denoted by the flow chart given below



5.4 OPTIMIZATION STUDY

This section mainly concentrates on how the optimization algorithm is mainly used in selecting the optimum or appropriate modes of operation during the bus operation. The algorithm that is used for this purpose is the ECMS algorithm (Equivalent consumption minimization strategy). This algorithm makes use of the following equation too determine values

$$\textbf{EC = Fuel + } \lambda \textbf{*Power of battery}$$

Where

- EC – Equivalent consumption
- Fuel – Fuel consumed
- λ – Correction Factor
- Power of battery –Battery power used up

With this equation of equivalent consumption all feasible points can be searched in the grid to provide us with the required results.

`

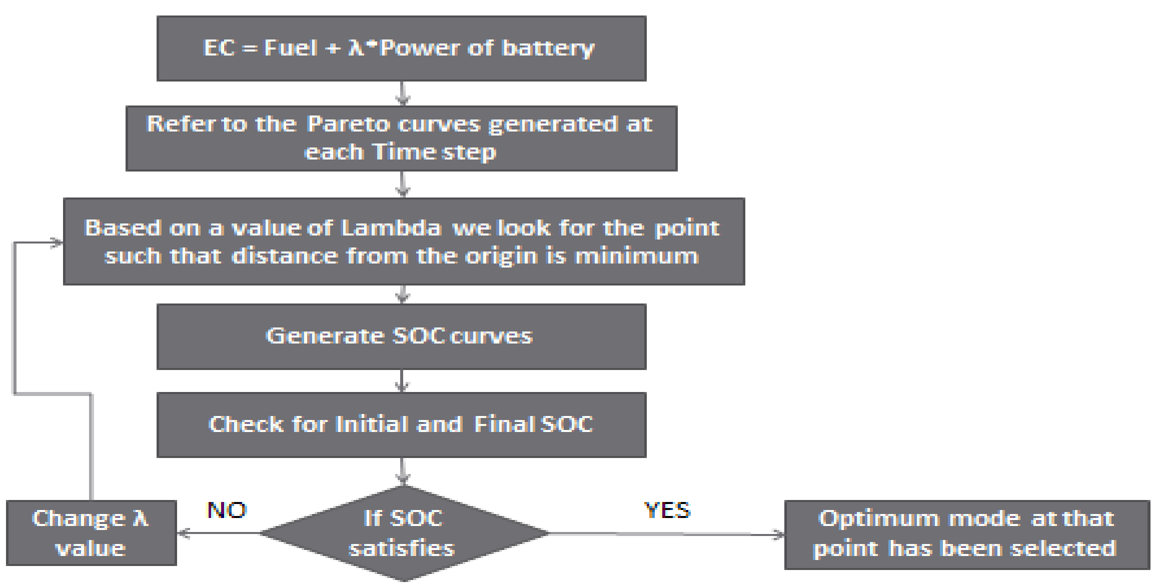
During the drive cycle the values of the Equivalent consumption can be calculated by using the fuel consumption and the batter power. At any particular time the characteristics of the engine and the motor are determined by minimizing the equivalent consumption and comparing those values with the Pareto curves that provide us with the lookup tables.

The value of the correction factor λ is chosen based on the values of the SOC. The correction factor λ chosen s considered to be correct if

$$|SOC_i - SOC_f| = 0 \text{ (within some tolerance)}$$

The SOC values that are chosen for the this project are at 60% the values of lambda are then calculated by a simple root finding algorithm leading us different values of lambda until the end SOC value is accurate within ±4% accuracy .

This process is repeated for all the values of times steps and the modes for each vehicle speed and torque are determined which together give us the optimum fuel efficiency of the bus. When the values of any of the parameters like FR , engine motor characteristics etc. change it leads to generation of new set of pareto curves after which the ECMS algorithm is performed again



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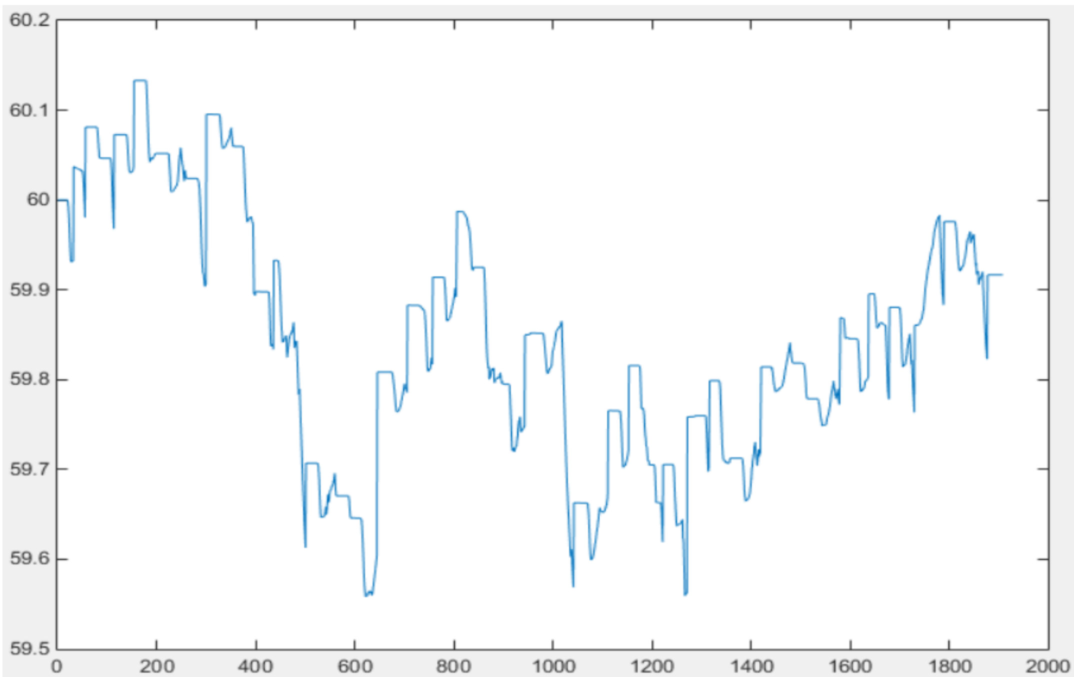
5.5 DISCUSSION OF RESULTS

The optimal control subprogram finally gives us the mileage (mpg) of the bus for the given drive cycle. Several runs performed were performed in the optimal control which produced the SOC curves.



- The graph above denotes the SOC curve. This run gave an mpg of 1.3411. The graph also shows that the battery use has been very negligible (about 0.5%). This is a result of the battery capacity being too high than what was required for the operation of the bus

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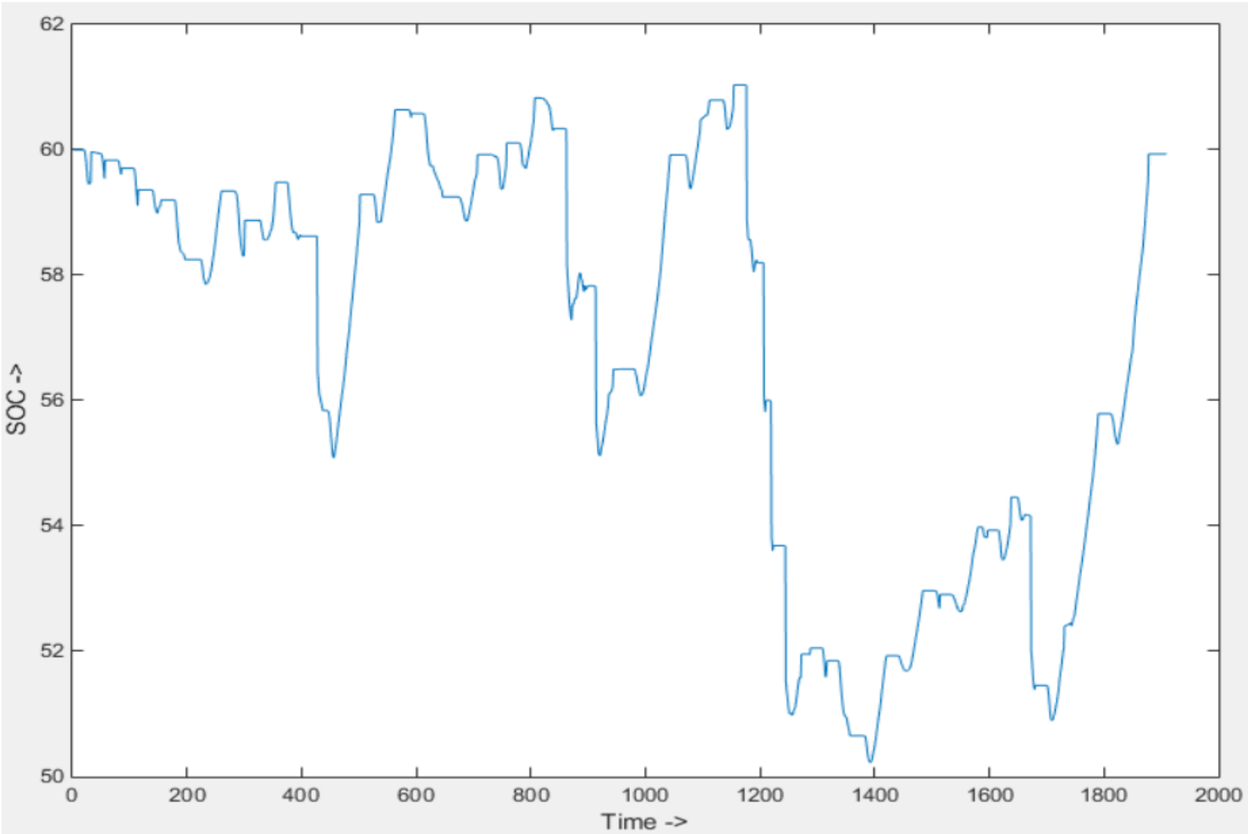

- The above graph is for another run of the optimal control wherein the battery usage has reached 10%. The corresponding mpg values obtained for this run was close to 6.1348.

- The graph shows that there is more optimal usage of the battery as well as the engine for powering the bus

- Based on multiple runs of the optimal control the mpg using the 3 systems combined was coming in the range of 5-7 mpg. This is almost 80% more efficient than a common transit bus which uses diesel engine alone

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6. SYSTEM INTERGRATION STUDY

The main objective of system integration is to obtain a maximum mpg value that is optimum after considering the results obtained from the individual subsystems

The system integration is done by taking in the values from the all the three subsystems and finding the optimum values based on these three system. It is done by using the fmincon function in matlab

The inputs to this fmincon function are the following

- N_s , N_p values
- FR values

- The value of Ns determines the voltage
- The value of N_p determine the battery capacity
- FR value determines coupler mass

All the three values also contribute to the total mass of the system and thus varying these values will change the mass of the bus which in turn will affect the mpg values.

The results that were obtained from the individual subsystems were

- From the battery subsystem we get the values of Ns and Np
    - **N_s=75**
    - **N_p=27**

- The value of FR value from the torque coupler
    - **FR = 8**
    - **Mass of coupler = 27.46 kg**

On running the final code of the optimal control along with the other subsystems the following results have been obtained for the transit bus operating in the Orange county drive cycle using a given engine and motor characteristics

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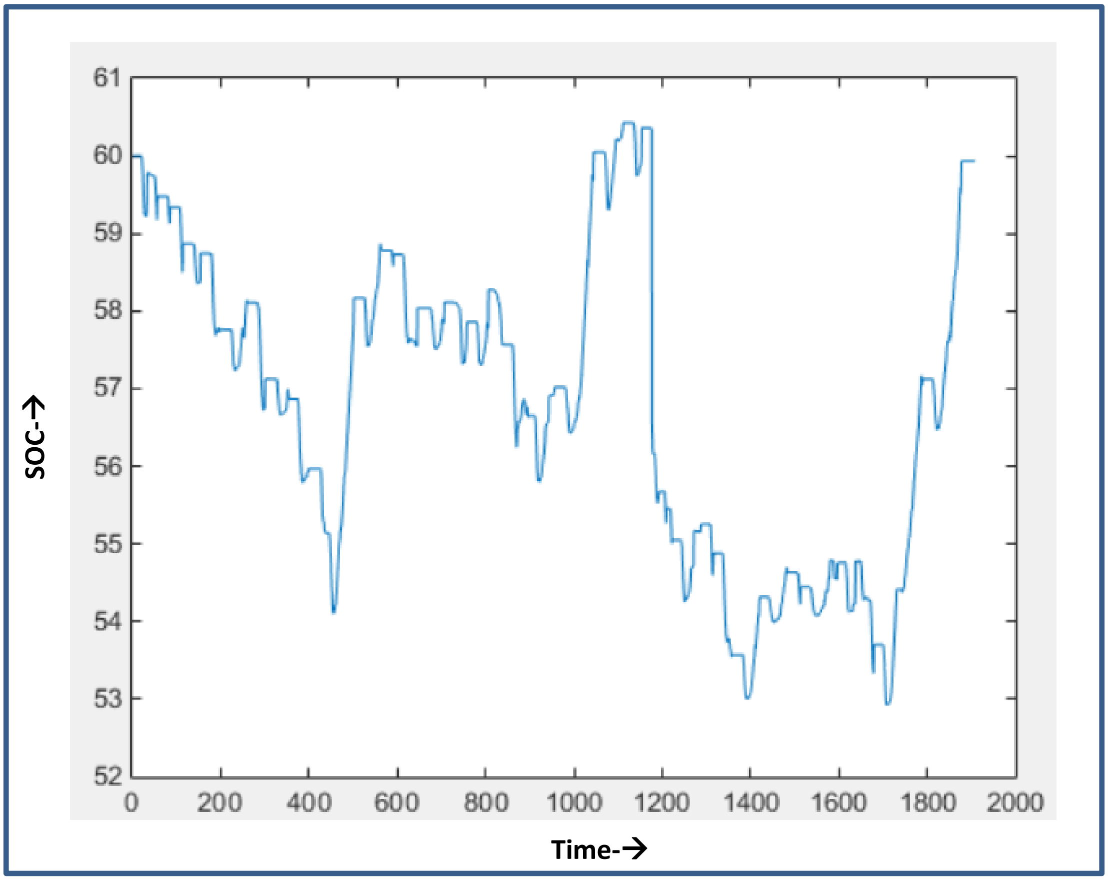
- MPG = 7.8475
- N_s = 22
- N_p = 12
- F.R = 8.48

The SOC curve after the system integration was performed is shown below. The curve shows that after the optimal control is performed the SOC curve reaches back to the required values . The y-axis range of the SOC curve shows that there has been a 10% usage of the battery for powering the bus when all the three subsystems are operated together .



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[5] Spur Gear Optimization By Using Genetic Algorithm by Yallamti Murali Mohan, T. Seshaiah, Department of Mechanical Engineering, QIS College of Engineering Technology, Ongole, Andhra Pradesh
[6] Gear Tooth Strength Analysis by Dr. W.H. Dornfeld, Fairfield University
[7] Gear Design equations and formulae by Engineers Edgeinvolute Spur Gear Design, IIT Madras, India

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[8] ASM Aerospace Specification Metals Inc. for Material data about 4130 Chrome Moly Steel
http://asm.matweb.com/search/SpecificMaterial.asp?bassnum=m4130r
[9] MakeItFrom.com for material properties about Grey Cast Iron and Carbon Steels
http://www.makeitfrom.com/
[10] The Engineering Toolbox website for material data, Ultimate tensile stress and Endurance limit for various materials used in the project.
http://www.engineeringtoolbox.com/

OPTIMAL CONTROL SUBSYSTEM

[1] Design of hybrid-electric vehicle architectures using Auto-generation of feasible driving modes by Alparslan Emrah Bayrak , Yi Ren , Panos Y. Papalambros
[2] Equivalent fuel consumption optimal control of a series hybrid electric vehicle by J-P Gao, G-M G Zhu, E G Strangas and F-C Sun
[3] Configuration Analysis for Power Split Hybrid Vehicles with Multiple Operating Modes by Xiaowu Zhang Chiao-Ting Li Dongsuk Kum Huei Peng Jing Sun
[4] Modeling and Control of a Power-Split Hybrid Vehicle by Jinming Liu and Huei Peng

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8. APPENDICES

8.1 APPENDIX – A (BATTERY SUBSYSTEM)

MATLAB CODE

Main Program

```
% Design Optimization Project
% Battery Subsystem
% Main Program
% x = [Pbat Preg Ns Np]; %Design Variables assigned to a vector x
% algType = 'active-set';
algType = 'interior-point';
% algType = 'sqp';
options  = optimset('Display','iter','MaxIter',1000, ...
   'MaxFunEvals',3000,'Algorithm',algType, ...
   'PlotFcns',@optimplotfval,'AlwaysHonorConstraints','bounds');
lb = [50000 -12000 50 10]; %Lower bound of decision variables
ub = [150000 -3000 80 25]; %Upper bound of decision variables
x0 = [90000 -10000 80 15]; %Initial Guess
X0 = scalFunc(x0,lb,ub,1); %Scaling function
LB = zeros(size(X0));
UB = ones(size(X0));
A = [];
b = [];
Aeq = [];
beq = [];
[xopt,fval,exitflag,output,lambda,grad,hessian] = fmincon('objFunc', ...
   X0,A,b,Aeq,beq,LB,UB,'conFunc',options);
xoptimal = (scalFunc(xopt,lb,ub,2));
```

Constraint Function

```
% Design Optimization Project
% Battery Subsystem
% Constraints Function
function [g,h] = conFunc(x)
%Initialization of Design Parameters
% x = [Pbat Preg Ns Np]; %Design Variables assigned to a vector x

lb = [50000 -12000 50 10]; %Lower bound of decision variables
ub = [150000 -3000 80 25]; %Upper bound of decision variables
X = scalFunc(x,lb,ub,2); %Scaling function
Pbat = X(1);
Preg = X(2);
Ns = X(3);
```

`

```
Np = X(4);
Vbat = 7.2; %Nominal cell voltage
Cmax = 1; %Maximum cell discharge rate
Cmin = -1; %Minimum cell discharge rate
Icmin = -2; %Minimmum charging current
Idmax = 5.5; %Maximum discharge current
Q = 5; %Battery capacity
M = 150000; %Mass of the battery pack
mbat =75;
SOCmin = 0.3; %Minimum SOC fixed at 30%
SOCmax = 0.9; %Maximum SOC fixed at 90%
SOCrange = (SOCmin:0.1:SOCmax)';
% SOCrange = 0.3;
rc = -8000;
w = zeros(length(SOCrange),1);
w(SOCrange<=0.8) = 1;
id = (SOCrange>0.8&SOCrange<=0.9);
w(id) = 9-10*SOCrange(id);
Voc = Ns*2*(3.6061 + 0.7737*SOCrange - ...
   0.9123*SOCrange.^2 + 0.6771*SOCrange.^3);
R = 0.01*(Ns/Np)*(0.0486 + 1.675*SOCrange ...
   - 5.190*SOCrange.^2 + 4.2533*SOCrange.^2);

%Equality Contraints
h = [];
%Inequality Constraints
g = [Pbat - Ns*Vbat*Np*Cmax*Q ; (Voc - sqrt(Voc.^2 - 4*R*Pbat))/2 ...
   - Ns*Vbat ; (Voc - sqrt(Voc.^2 - 4*R*Pbat))./(2*R) - Np*Idmax ; ...
   Np*Icmin + (Voc - sqrt(Voc.^2 - 4*R*Preg))./(2*R) ; ...
   (Voc - sqrt(Voc.^2 - 4*R*Pbat))./(2*R*Q*Ns) - Cmax ; Cmin - ...
   (Voc - sqrt(Voc.^2 - 4*R*Preg))./(2*R*Q*Ns) ; Ns*Np*mbat - M ; ...
    Preg - w*rc];
end
```

Objective Function

```
% Design Optimization Project
% Battery Subsystem
% Objective Function
function [f] = objFunc(x)
lb = [50000 -12000 50 10]; %Lower bound of decision variables
ub = [150000 -3000 80 25]; %Upper bound of decision variables
X = scalFunc(x,lb,ub,2); %Scaling function
```

`

```
f = -(X(1) + X(2)); %Objective function
end
```

Scaling Function

```
% Design Optimization Project
% Battery Subsystem
% Scaling Function
function X = scalFunc(x,l,u,type)
%This function scales the vector x that contains the design variables
%depending upon the bounds which are represente by l and u. Type is the
%flag variable where 1 is to scale and 2 is to unscale. This is done
%to maintain the vector of the same dimension
for i = 1:1:4
    if type == 1
        X(i) = (x(i) - l(i))/(u(i) - l(i));

    else
        X(i) = l(i) + x(i)*(u(i) - l(i));
    end
end
```

Plots

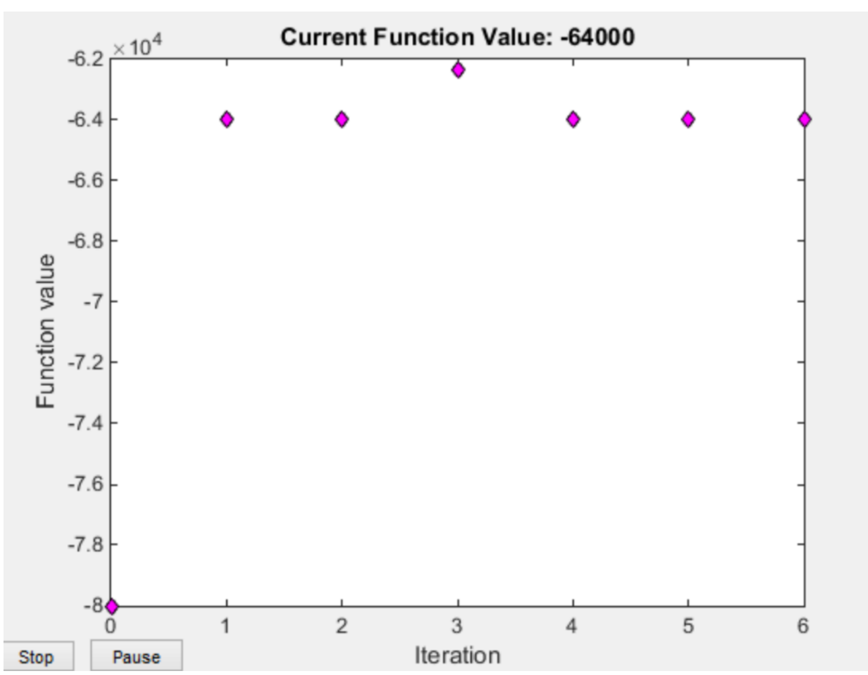


Figure 3 Active Set Algorithm

`

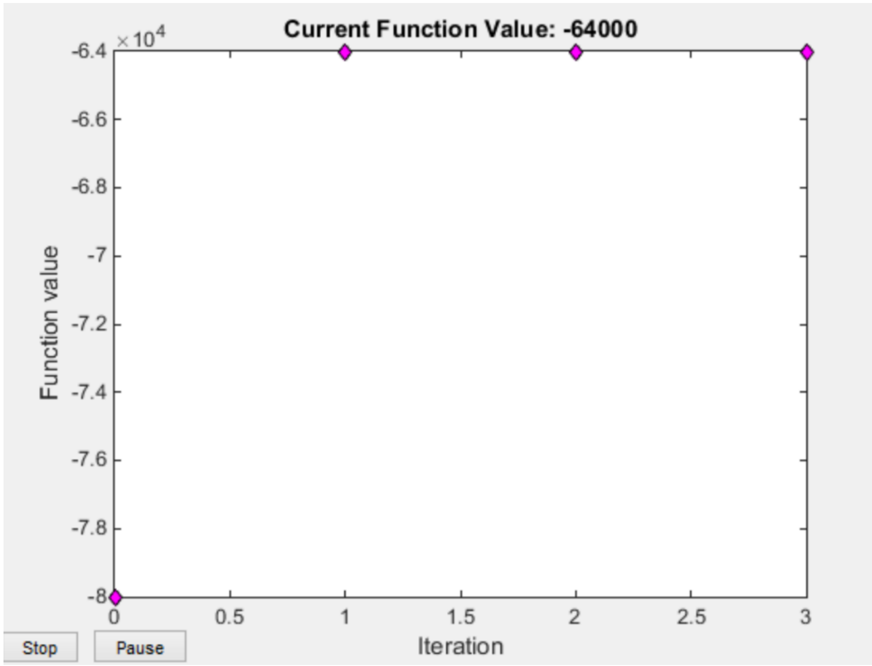


Figure 4 SQP Algorithm

The curve fitting figure is showing below.

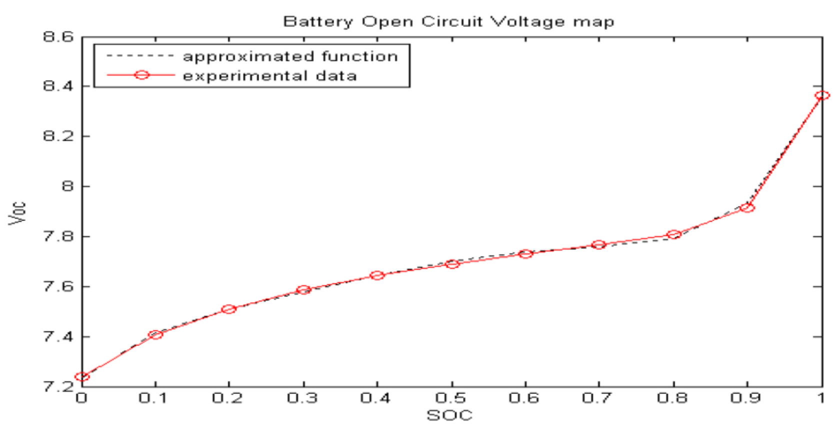


Figure 5 Curve Fitting for Voc wrt SOC

`

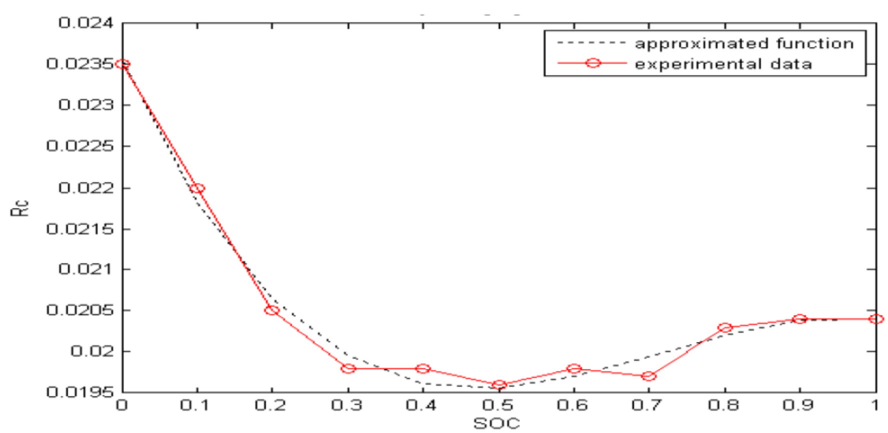


Figure 6 Curve Fitting for R wrt SOC

`

8.2 APPENDIX – B (MECHANICAL COUPLER SUBSYSTEM)

1. Monotonicity Analysis

G1 → $T - \sigma_{TU} * b * (0.154 * r - 0.456) \leq 0$

G2 → $T * G - \sigma_{TU} * b * (0.154 * r * G - 0.456) \leq 0$

G3 → $\sqrt{\frac{E*(T*\sqrt{r}-\sqrt{a})*(G+1)}{2*(1-\mu^2)*\cos\emptyset*b*\sqrt{\pi^3}*\sqrt{r^5}*G*\sqrt{(1+G^2)}}} - \sigma_{\mathrm{endl}} \leq 0$

G4 → -r < 0

G5 → -b < 0

G6 → -G + 1 < 0

G7 → b*(G + 1) – r*(G) = 0

MONOTONICITY TABLE

	r	B	G
f(x)	+	+	-
G1	-	-	+
G2	-	-	+
G3	-	-	+
G4	+		
G5		+	
G6			+
G7	+	-	+

2. MATLAB CODES

Objective Function Code:

```
function[f] = PROJFUN(x)
% Density of the gear material i.e. 4130 Chrome Moly Steel used = % 0.78*10^-5
Kg/mm^3
f = pi*0.78*(10^-5)*x(1)*x(2)^2*((x(3)^2 + 2));
end
```

`

```
% b = Width of the driving gear, denoted as variable x(1), mm
% r = Radius of the driving gear, denoted as variable x(2), mm
% G = Gear Ratio between the gears, denoted as variable x(3), mm

%All the dimensions of length are in 'mm'

%function f defines the weight of the mechanical torque coupler
```

Nonlinear Constraints Code:

```
function[a,b] = PROJNONLCON(x)

%Inequality constraints
a = [1600000 - 620*5.25*x(1)*(0.154*x(2) - 0.456);
1600000*x(3) - 620*5.25*x(1)*(0.154*x(2)*x(3) - 0.456);
sqrt((206000*(1600000*sqrt(x(2)) - 45.75)*(x(3) + 1))/(0.9*pi^1.5*x(1)*x(2)^2.5*x(3)*sqrt(x(3)^2 + 1))) - 480];

% 1600000 - 620*5.25*x(1)*(0.154*x(2) - 0.456) -->
% Constraint for maximum bending stress in the input gears coupled with the Engine and motor outputs

% 1600000*x(3) - 620*5.25*x(1)*(0.154*x(2)*x(3) - 0.456) -->
% Constraint for maximum bending stress in the output gear of the mechanical coupler

% sqrt((206000*(1600000*sqrt(x(2)) - 45.75)*(x(3) + 1))/(0.9*pi^1.5*x(1)*x(2)^2.5*x(3)*sqrt(x(3)^2 + 1))) - 480 -->
% Constraint for maximum contact stress between the input gear and the output gear

%Equality constraints

b = [x(1)*x(3) + x(1) - x(2)*x(3)];

% x(1)*x(3) + x(1) - x(2)*x(3) -->
% Design constraint of spur gears, ratio of width of the gear to its radius is equal to the ratio of gear ratio to sum of gear ratio and 1
```

`

```
end
```

FMINCON SOLVER CODE:

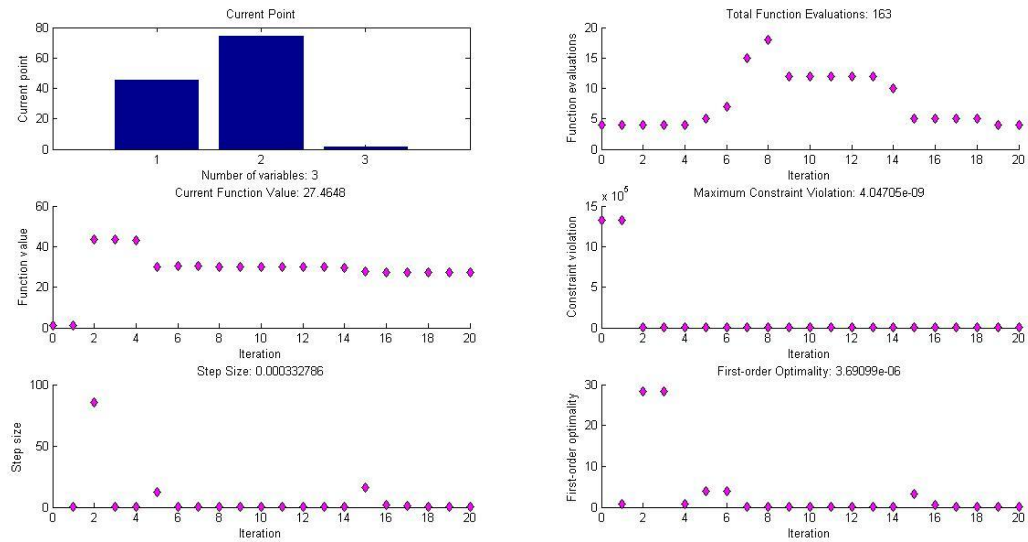
```
function PROJOPT
clear all
clc
A=[ ];
b=[ ];
Aeq = [ ];
beq = [ ]; % matrix/vectors for defining linear constraints (not used)
lb = []; % lower bounds on the problem
ub = []; % upper bounds on the problem (not used)
xopt = fmincon('PROJFUN',[50,50,3],A,b,Aeq,beq,lb,ub,'PROJNONLCON');
[xopt,fval,exitflag,output,lambda,grad,hessian] =
fmincon('PROJFUN',[50,50,3],A,b,Aeq,beq,lb,ub,'PROJNONLCON');
fprintf('xopt = \n');disp(xopt)
fprintf('fval = \n');disp(fval)
fprintf('exitflag = \n');disp(exitflag)
fprintf('output = \n');disp(output)
fprintf('lambda = \n');disp(lambda)
fprintf('grad = \n');disp(grad)
fprintf('hessian = \n');disp(hessian)

% fprintf('lambda =');disp(lambda)
% fprintf('gradient =');disp(grad)
% fprintf('Hessian =');disp(hessian)

end
```

RESULTS AND PLOTS

1. Starting Point: [25 25 1]

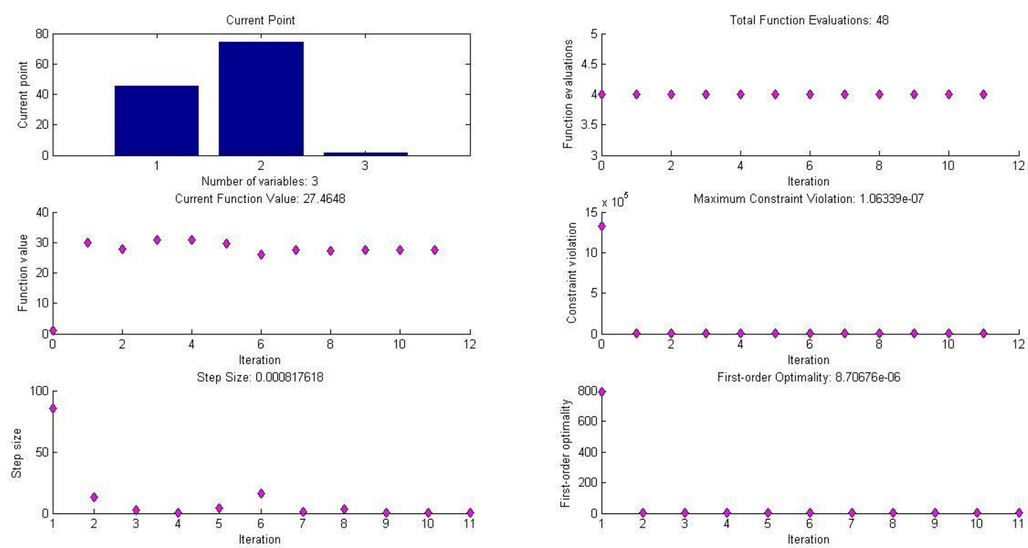
i. Algorithm: Interior Point

Solution: b = 45.439; r = 74.509; G = 1.56; W = 27.4648



ii. Algorithm: SQP

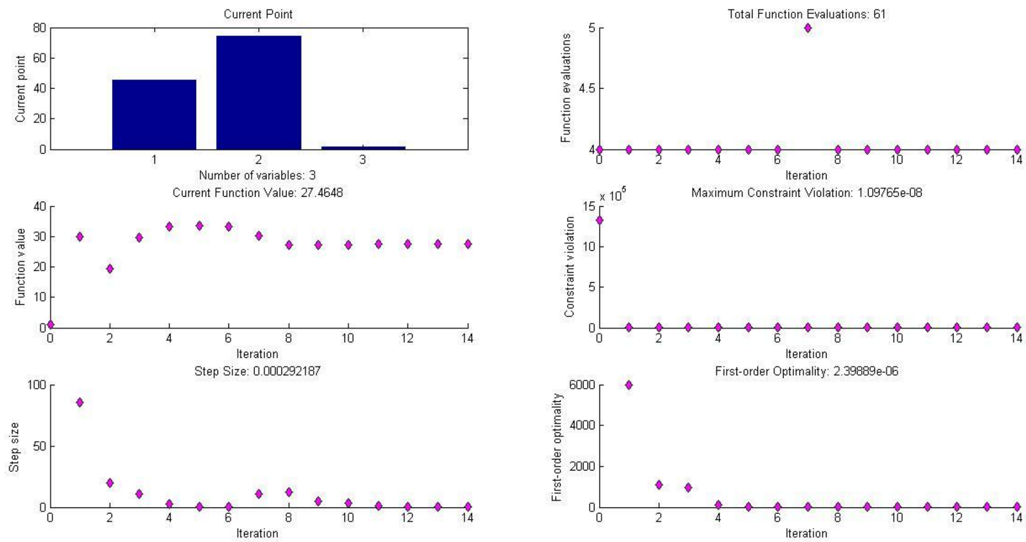
Solution: b = 45.439; r = 74.509; G = 1.563; W = 27.4648



iii. Algorithm: Active Set

Solution: b = 45.439; r = 74.509; G = 1.56; W = 27.4648

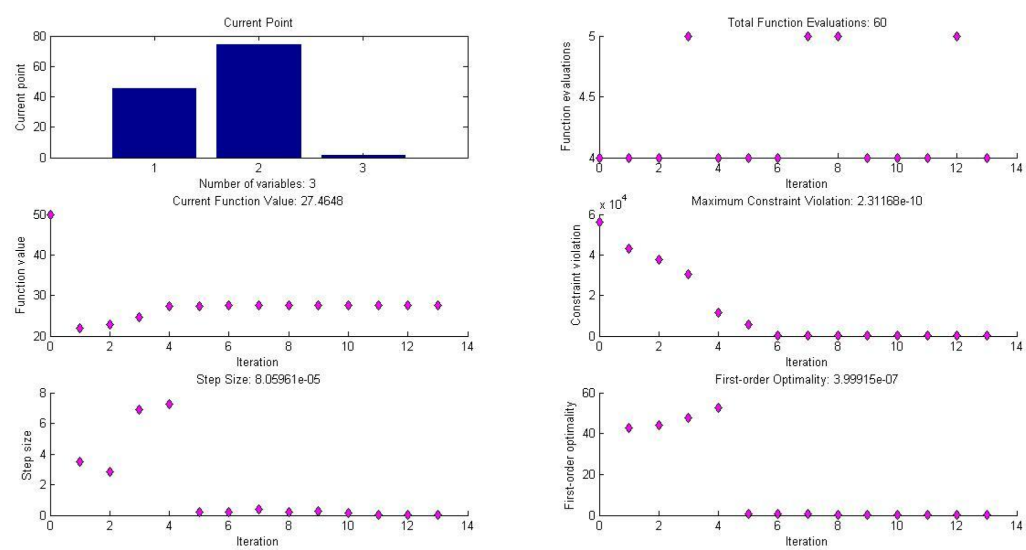
`



2. Starting Point: [25 25 1]

i. Algorithm: Interior Point

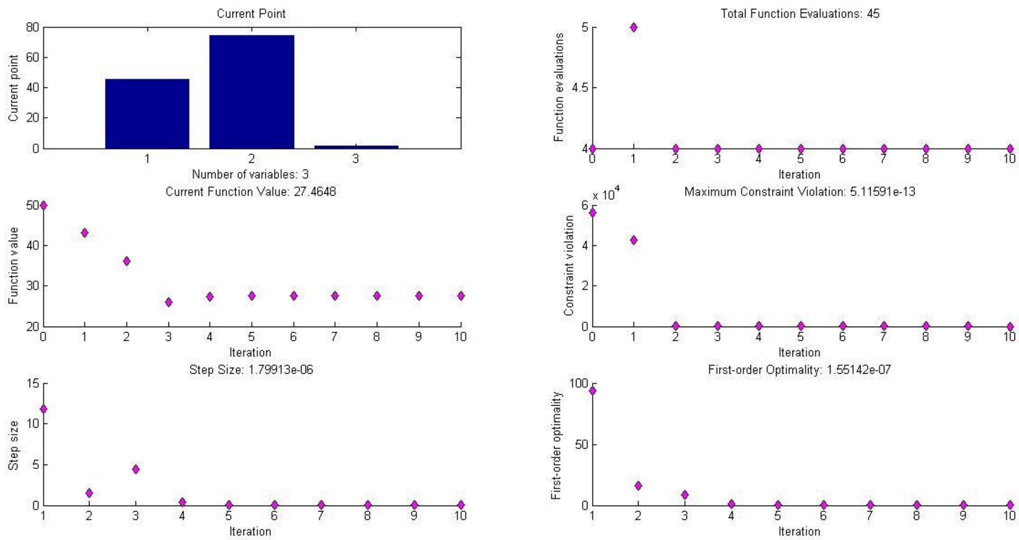
Solution: b = 45.439; r = 74.509; G = 1.56; W = 27.4648



ii. Algorithm: SQP

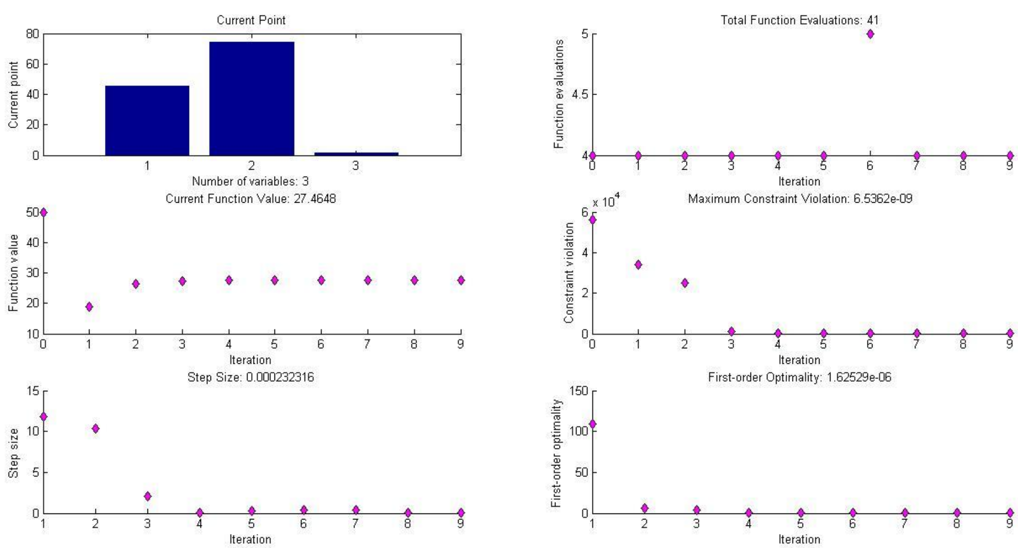
`

Solution: b = 45.439; r = 74.509; G = 1.56; W = 27.4648



iii. Algorithm: Active Set

Solution: b = 45.439; r = 74.509; G = 1.56; W = 27.4648

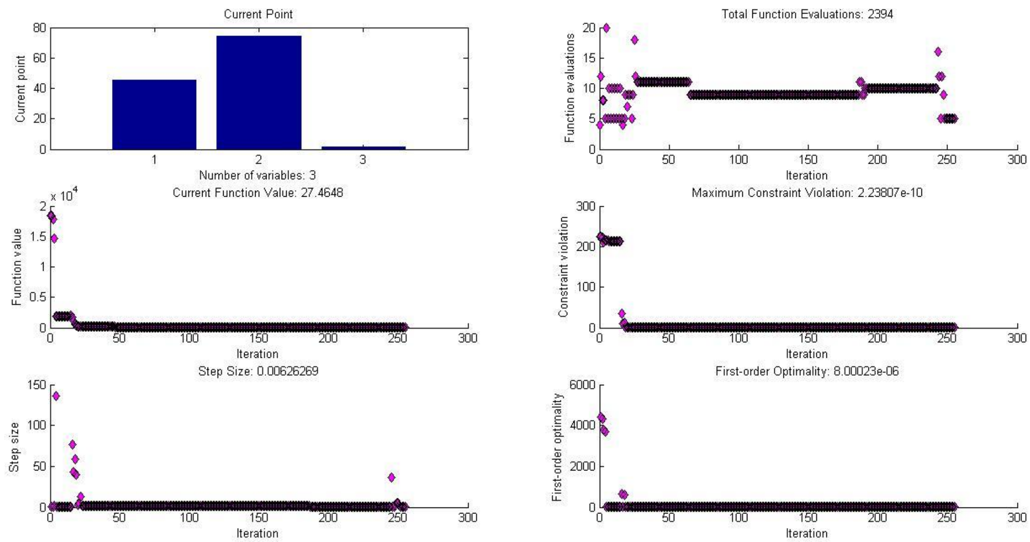


3. Starting Point: [225 225 8]

i. Algorithm: Interior Point

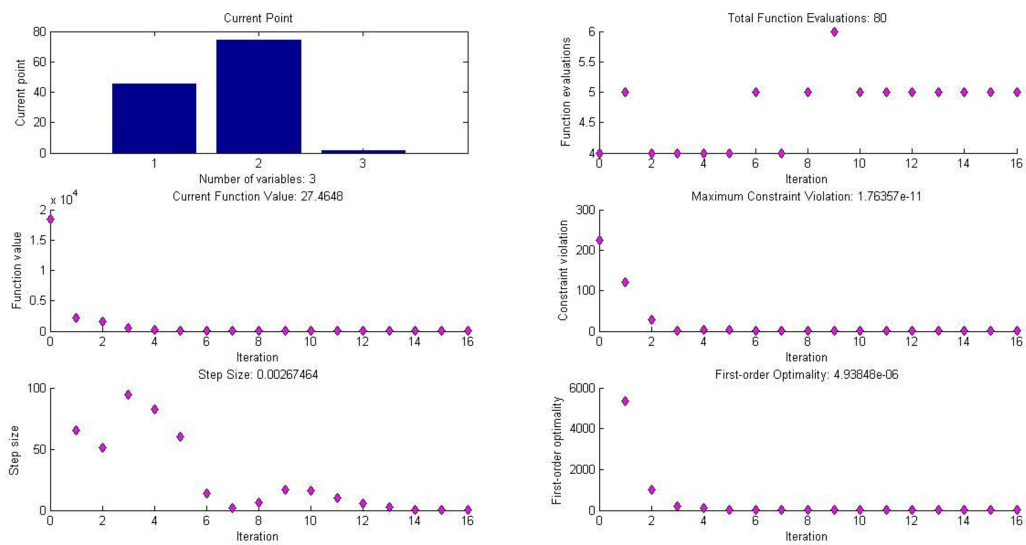
Solution: b = 45.439; r = 74.509; G = 1.56; W = 27.4648

`



ii. Algorithm: SQP

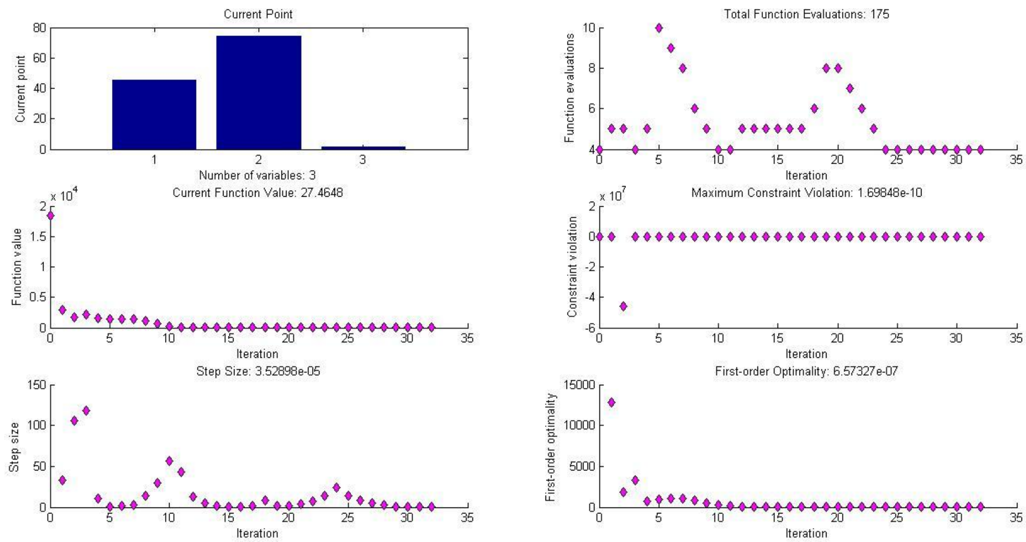
Solution: b = 45.439; r = 74.509; G = 1.56; W = 27.4648



iii. Algorithm: Active Set

Solution: b = 45.439; r = 74.509; G = 1.56; W = 27.4648

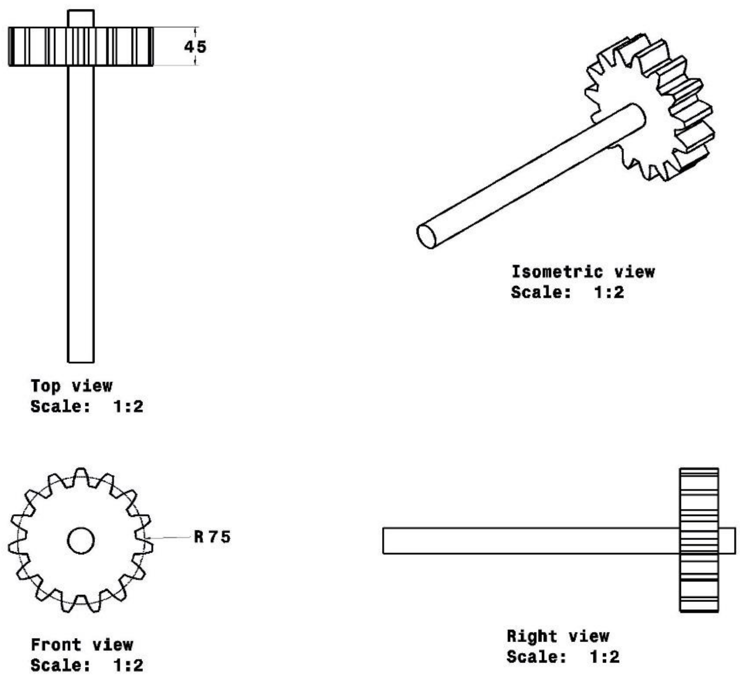
`



CAD MODEL AND 2D DRAWINGS

3D CAD Model of the Optimized Input Gear

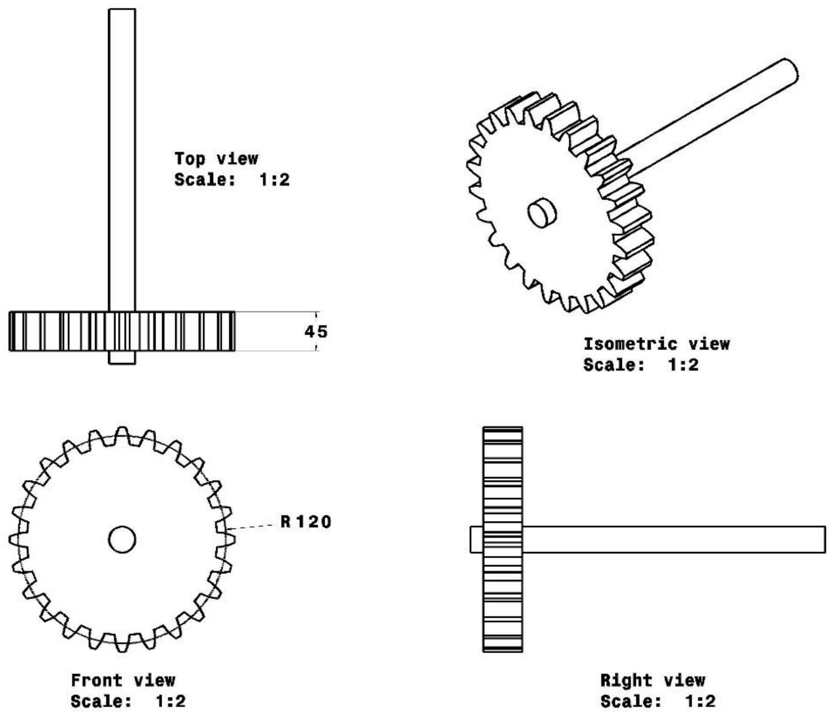
`



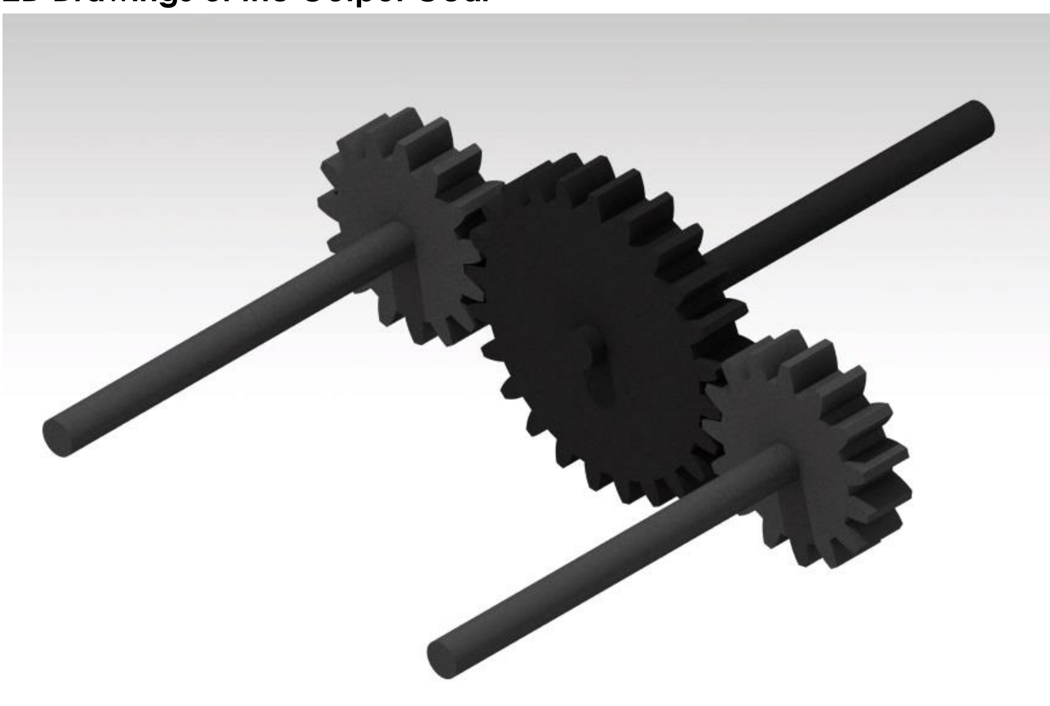
2D Drawings of the Input Gear

3D Model of the optimized Output Gear

`

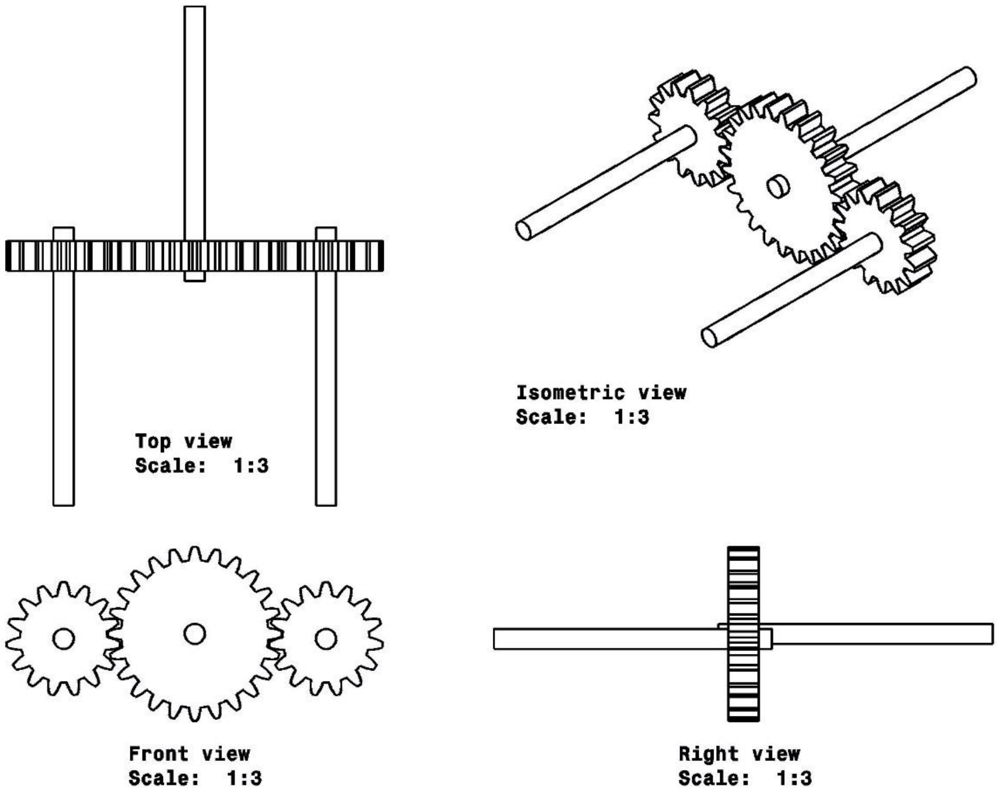


2D Drawings of the Output Gear



3D Model of the Mechanical Coupler

`



2D Drawings of the Mechanical Coupler

8.3 APPENDIX – C (OPTIMAL CONTROL SUBSYSTEM)

Please contact Dr Yi Ren , Assistant Professor , Arizona State University