DESIGN OPTIMIZATION OF BRAKE DISC GEOMETRY

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ABSTRACT

The motivation of this project is to determine the optimum dimensions of the brake disc for a four-wheeler vehicle using MATLAB and ANSYS. These dimensions include the disc inner radius, outer radius and thickness. Structural, modal and thermal load cases for emergency braking conditions are individually considered to determine these dimensions. The optimization objective is to minimize the brake disc volume, whereas the other objectives are to minimize the stress, temperature and maximize the first natural frequency of the disc. These goals are accomplished using optimization algorithms in MATLAB and the results are correlated with the values obtained from ANSYS. Finally, system optimization is performed using MOGA by integrating all the load cases.

ACKNOWLEDGMENTS

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1   Design Problem Statement

Primary objective: To minimize the brake disc volume for emergency braking conditions

Secondary objectives:

- Minimize the maximum stress in the brake disc

- Maximize the first natural frequency of the brake disc

- Minimize the maximum temperature in the brake disc

1.1   Subsystem 1: Structural Analysis

Team Member: Swapnil Nimse

The brake disc has to sustain the pressure from the hydraulically actuated brake pads during sudden braking conditions. Stresses are induced due to friction between the brake pads and the disc. The disc also experiences centrifugal body forces due to its rotation. Resultant stresses generated due these forces can lead to material failure. Therefore, it is of prime importance to make sure that the stresses in the disc are minimized.

1.2   Subsystem 2: Modal Analysis

Team Member: Abhijeet Durgude

Free modal analysis is performed to ensure that the disc's first natural frequency is higher than the engine firing frequency. This guarantees that the disc does not experience failure due to resonance.

1.3   Subsystem 3: Thermal Analysis

Team Member: Sharan Kishore

Braking in a vehicle takes place due to friction between the brake pads and the rotor disc. This leads to heat flux generation in the disc which consequently results in increase in its temperature and thermal stresses. Emergency braking conditions induce high temperatures that damage the contact surfaces. It is therefore essential to minimize the temperature to prevent disc wear and tear.

1.4   Subsystem 4: MATLAB Programming

Team Member: Aditya Vipradas

Subsystem and system level optimization are performed by programming the entire procedure in MATLAB. This includes the use of optimization algorithms like *interior-point method (fmincon)*,

and Multi-Objective Genetic Algorithm (MOGA), plotting the Pareto curves for subsystem and system level optimization and generating results for validation purposes.

The design optimization methodology flowchart is as shown in Figure 1.
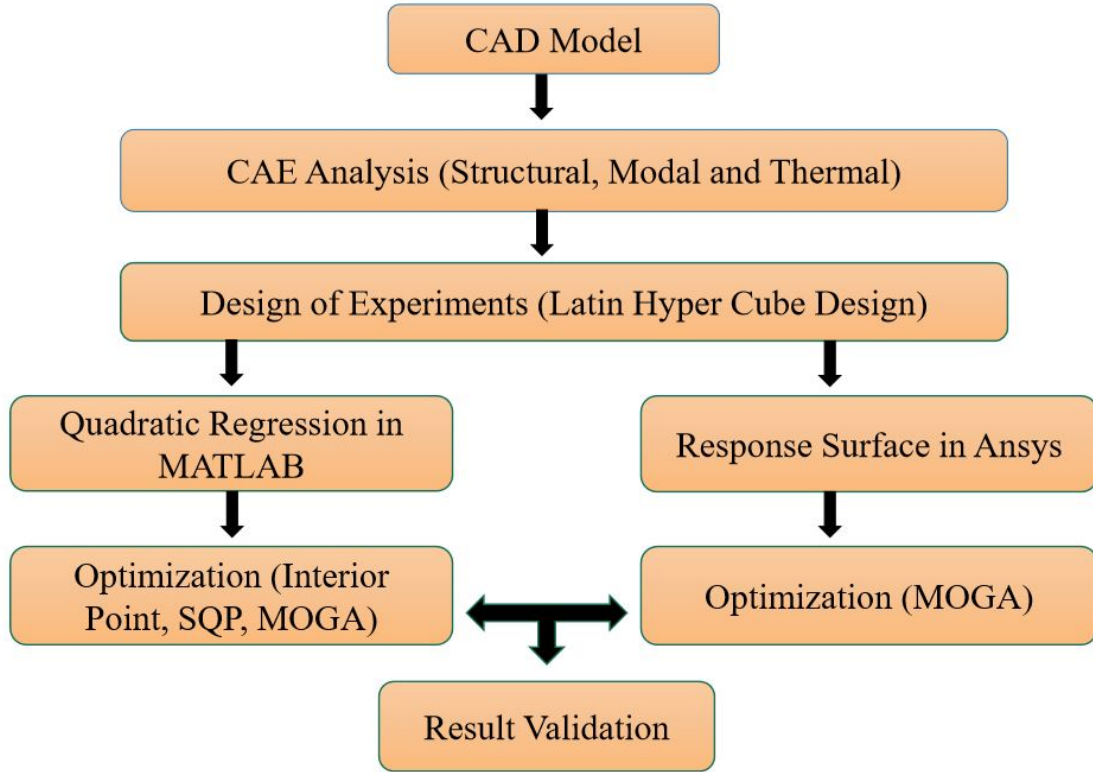
Figure 1: Design optimization process flowchart

2   Nomenclature

V: Volume (mm^3)

α: Regression coefficients for volume

S: Stress (MPa)

β: Regression coefficients for stress

F: First natural frequency (Hz)

γ: Regression coefficients for frequency

T: Temperature (^oC)

δ: Regression coefficients for temperature

*P*1: Inner disc radius (*mm*)

*P*2: Outer disc radius (*mm*)

*P*3: Disc thickness (*mm*)

d: Braking distance (*m*)

t_e: Braking time (*s*)

u: Initial vehicle velocity (*kmph*)

m: Vehicle mass (*kg*)

ν: Coefficient of friction

w: Disc rotational velocity (*rad/s*)

R_{avg}: Average of P1 and P2 (*mm*)

KE: Vehicle kinetic energy (*J*)

F_b: Total braking force on the disc (*N*)

P: Pressure acting on the disc due to one brake pad (*Pa*)

A_p: Brake pad area (*mm^2*)

A_{sp}: Brake pad swept area (*mm^2*)

q: Heat flux (*W/m^2*)

T_{amb}: Ambient temperature (*^oC*)

3 Mathematical Model

The brake disc inner radius (P1), outer radius (P2) and thickness (P3) are the design variables in this optimization study. Initially, structural, modal and thermal analyses are performed in ANSYS. For the assumed geometric constraints, Design Of Experiment (DOE) points are generated. Mathematical model is then generated by performing a 2nd order regression analysis on these DOE points to obtain the volume, stress, frequency and temperature quadratic functions. The optimization problem is designed as follows:

Primary objective:

$$Minimize : f1 : V = \sum_{n=1}^{n=3} \alpha_n x_n + \sum_{n=4}^{n=6} \alpha_n x_n^2 + \alpha_7; (R_{volume}^2 = 0.98) \tag{1}$$

Secondary objectives:

$$f2 : Minimize : S = \sum_{n=1}^{n=3} \beta_n x_n + \sum_{n=4}^{n=6} \beta_n x_n^2 + \beta_7; (R_{stress}^2 = 0.53)^1 \tag{2}$$

$$f3 : Maximize : F = \sum_{n=1}^{n=3} \gamma_n x_n + \sum_{n=4}^{n=6} \gamma_n x_n^2 + \gamma_7; (R_{frequency}^2 = 0.92) \tag{3}$$

$$f4 : Minimize : T = \sum_{n=1}^{n=3} \delta_n x_n + \sum_{n=4}^{n=6} \delta_n x_n^2 + \delta_7; (R_{thermal}^2 = 0.97) \tag{4}$$

Geometrical constraints for all subsystems:

$$g1 : -P1 \leq -66 \tag{5}$$
$$g2 : P1 \leq 90 \tag{6}$$
$$g3 : -P2 \leq -124 \tag{7}$$
$$g4 : P2 \leq 150 \tag{8}$$
$$g5 : -P3 \leq -5 \tag{9}$$
$$g6 : P3 \leq 27 \tag{10}$$

This is a multi-objective optimization problem. A multi-objective optimization algorithm can be readily solved using MOGA. But MOGA will not always give the best optimum value. Therefore, it is beneficial to convert the multi-objective problem into a single objective problem by assuming the rest of the objectives as constraints. In order to use single-objective optimization algorithms for this problem, it is thus necessary to provide upper or lower bounds on the secondary objective functions. This aids in plotting the desired Pareto curves. The bounds are determined by taking into consideration the brake disc failure.

Design constraints:

$$g7 : S \leq 14MPa \tag{11}$$
$$g8 : -F \leq -1200Hz \tag{12}$$
$$g9 : T \leq 400^oC \tag{13}$$

The sub-system level optimization problems are designed as follows:

3.1   Structural Analysis Model

$Min : f1, f2$

s.t.
$g1, g2, g3, g4, g5, g6$ and $g7$

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[1] 5th order polynomial fit gives a better R^2 value of 0.77. Kriging model may work better as well.

3.2 Modal Analysis Model

$Min : f1, -f3$

s.t.
$g1, g2, g3, g4, g5, g6$ and $g8$

3.3 Thermal Analysis Model

$Min : f1, f4$

s.t.
$g1, g2, g3, g4, g5, g6$ and $g9$

4 Model Analysis

The flowchart for optimization in ANSYS is shown in Figure 2. This figure pertains to the flowchart for structural analysis. Similar procedure is followed for modal and thermal analyses.
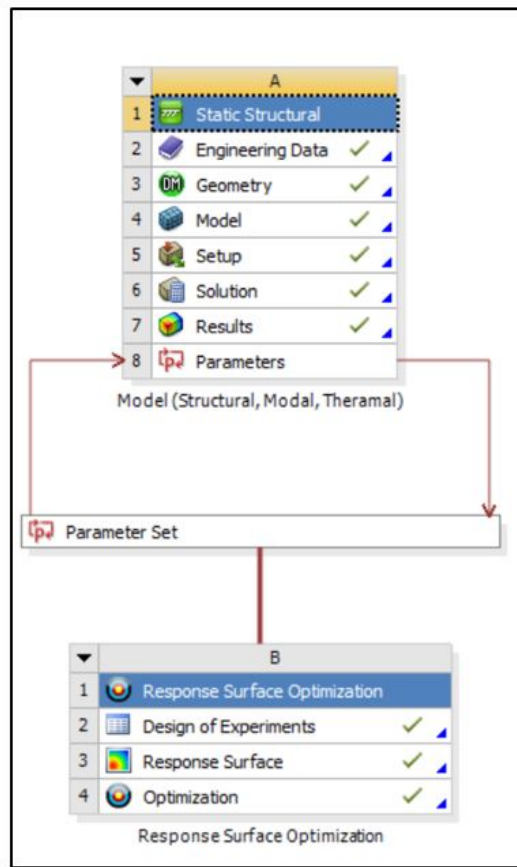
Figure 2: ANSYS Optimization Flowchart

4.1 FE Model

The brake disc geometry is prepared in ANSYS DesignModeler as shown in Figure 3. The initial values for P1, P2 and P3 are considered to be 75 mm, 125 mm and 25 mm respectively. These values are obtained from the literature survey by considering disc dimensions in different vehicles.
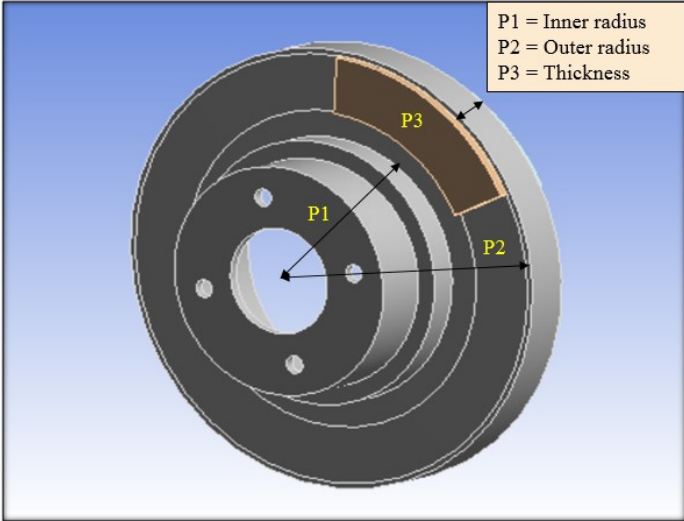
Figure 3: Disc Brake CAD Model

The CAD geometry is meshed using 6 mm sized tetrahedral quadratic elements as shown in Figure 4.
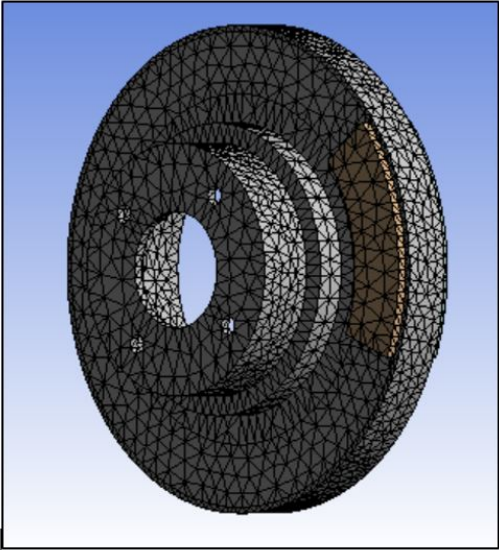
Figure 4: Disc Brake Mesh Model

The brake disc is made of gray cast iron. The material properties of gray cast iron are shown in Table 1.

Property	Value	Unit
Density	7200	kg/m^3
Young's modulus	110	GPa
Poisson's ratio	0.28	-
Thermal conductivity	52	W/m^oC
Specific heat	447	J/kg^oC

Table 1: Gray cast iron material properties

The assumptions made for performing the FEM analysis are as follows:

- The braking torque distribution between the front and rear axles is 70:30.

- Natural convection takes place due to the ambient air.

- The disc brake considered is of the solid type.

- Heat flux on the disc brake acts on both sides of the disc.

4.2 Structural Analysis

Considering the emergency braking distance of 10 m with stopping time of 5 seconds for a vehicle with initial velocity of 90 km/hr, the braking pressure required is calculated. This pressure will act as one of the boundary conditions for the simulation. It is assumed that 70% of the braking power is in the front axle of a four-wheeler vehicle. The force acting on a single disc brake is calculated by multiplying the front axle braking power by 0.5. Using this, the pressure on each pad is calculated by considering the coefficient of friction. Here, $d = 10m; t_e = 5s; u = 90kmph; m = 1500kg; \nu = 0.22; A_p = 3552mm^2$.

Calculating the disc angular velocity (ω),

$$\omega = u/R_{avg} = 250 rad/s \tag{14}$$

Evaluating the total braking force on the disc (F_b),

$$F_b = (KE * 0.5 * 0.7)/d = 16.406kN \tag{15}$$

Formulating the pressure exerted by one brake pad on the disc (P),

$$P = F_b/(2 * A_p * \nu) = 10.495MPa \tag{16}$$

Static structural analysis is performed on the aforementioned geometry due to the brake pad actuating load acting on the disc. The boundary conditions imposed on the brake geometry are as follows. See Figure 5.

- The inner circumference of the wheel hub has a revolute joint applied.

- The brake caliper pads are constrained in X and Z directions. They are kept free to move in the Y direction.

- The rotational velocity of 250 rad/s is applied on the disc at the wheel hub attachment point.

- Frictional contact is provided between the brake caliper pads and brake rotor.

- Actuating pressure of 10.496 MPa is applied on the brake pads.
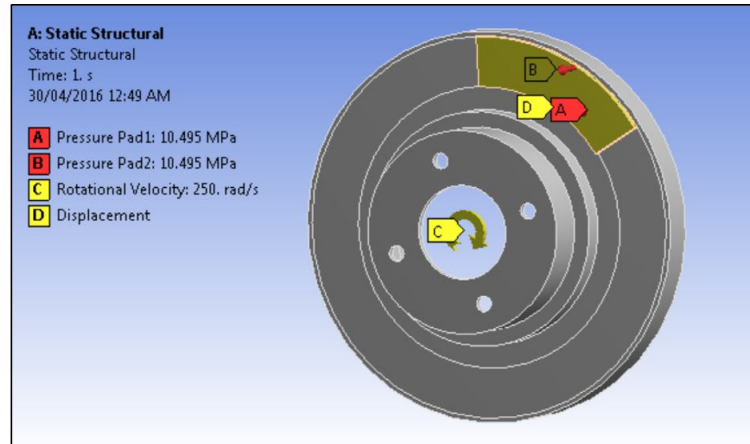
Figure 5: Static structural boundary conditions

The stress plot obtained is as shown in Figure 6. As observed, the maximum stress of 14.262 MPa is obtained for initial conditions on the design variables.

Figure 6: Stress plot for initial conditions on the design variables

4.3 Modal Analysis

Modal analysis is performed on the brake disc to determine its free natural frequency. The brake caliper pad geometry is suppressed while performing this analysis because the natural frequency of the brake disc is to be determined. Figure 7 shows the first natural frequency and its mode shape.
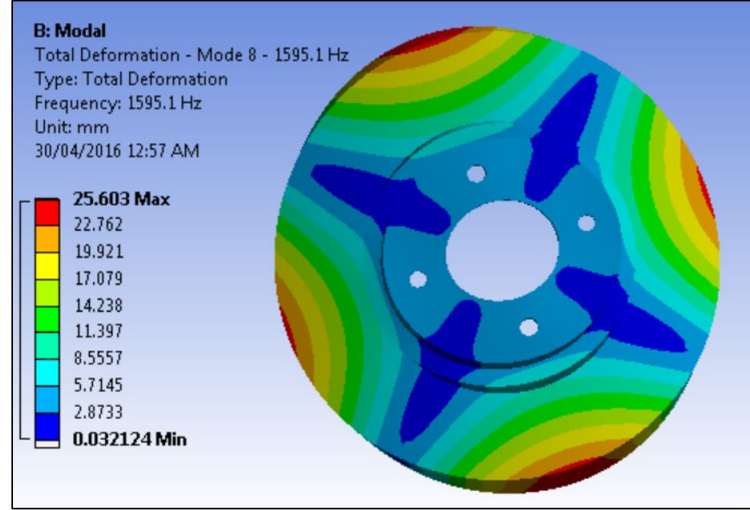
Figure 7: Mode shape plot for initial conditions on the design variables

4.4 Thermal Analysis

Transient thermal analysis is performed on the disc to observe the maximum temperature rise after the braking operation. The heat flux is calculated as shown below. It is assumed that 70% of the braking power is in the front axle of a four-wheeler vehicle. The total heat flux is also multiplied by 0.5 to get the flux generated by a single pad on the disc brake. Here, $t_e = 5s; A_{sp} = 0.021m^2; T_{amb} = 35^oC$.

Calculating the heat flux (q) generated on each face of the disc,

$$q = (KE * 0.5 * 0.7)/(t_e * A_{sp}) = 1.5395e6 W/m^2 \qquad (17)$$

Boundary conditions applied for the transient thermal analysis are as follows:

- Heat flux of $1.5395e6 W/m^2$ is applied on the swept area of both the pads while the direction of heat flow is towards the disc.

- Convection is applied on all the surfaces with the air film coefficient of $5W/m^2 K$ which is the default value for standard air.

- Initial temperature is kept at 35^oC.

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The analysis is performed for 5 s which is the braking time. The boundary conditions for this analysis are shown in Figure 8
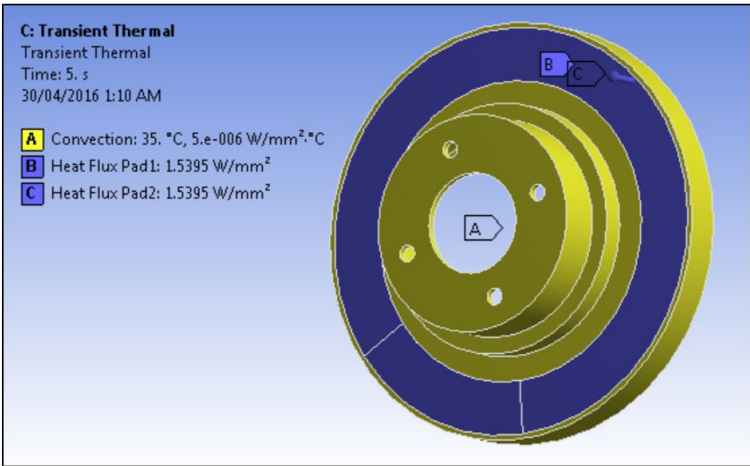
Figure 8: Transient thermal boundary conditions

The temperature plot obtained is as shown in Figure 9. As observed, the maximum temperature of 321^oC is obtained for initial conditions on the design variables.
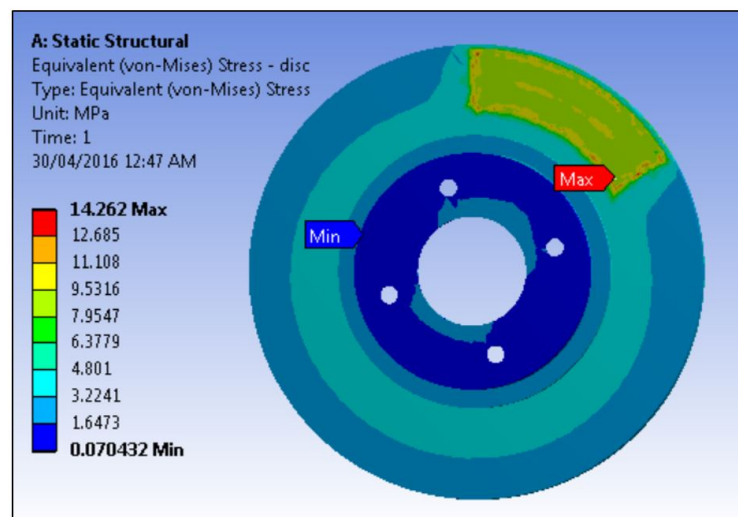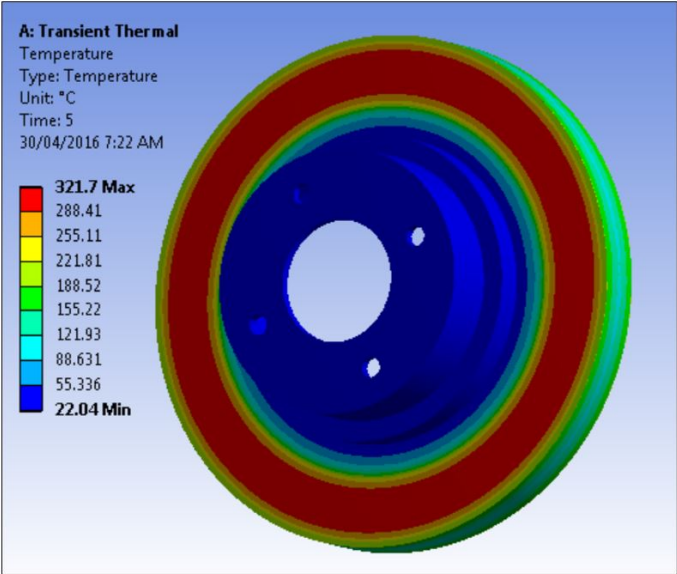

Figure 9: Temperature plot for initial conditions on the design variables

4.5 Design of Experiments

After the initial analysis of each subsystem, the relationship between the design variables and output response is determined. All of the input variables are quantitative and continuous

in nature. To obtain the accurate response surface, minimum number of design points from the given sample space are required. Latin Hypercube Sampling (LHS) technique with user defined sample points is used to create the response surface. The main advantage of LHS is that all the sample points are varying in nature. Unlike other sampling methods like Central Composite Design (CCD) and the full factorial methods, the number of simulations required for LHS remains constant even with an increasing number of parameters. After the design of experiments process, we obtain various combinations of input parameters and the corresponding response values. All the 50 DOE points are shown in Appendix A.

4.6   Response Surface

After DOE, a response surface is generated for all the input and output values using the least squares methodology. The data points are fitted with a standard 2nd order model. The points generated on the response surface are then used to perform the optimization. The goodness of fit plots for all the subsystems are shown below.

Figure 10: Goodness of fit plot for structural analysis

Figure 11: Goodness of fit plot for modal analysis
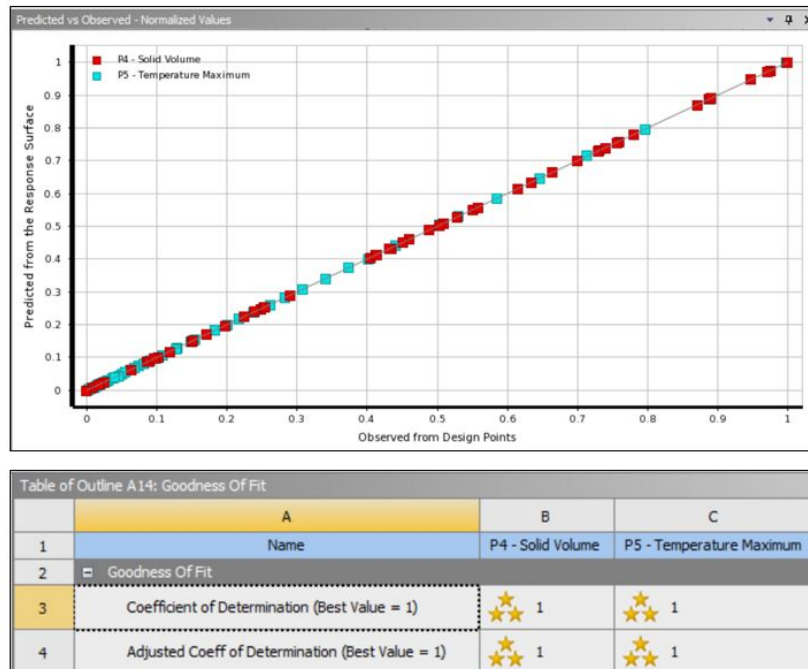
Figure 12: Goodness of fit plot for thermal analysis

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5   Optimization Study

5.1   Structural Analysis Optimization

For the subsystem 1 optimization problem, the upper bound on the stress constraint from Equation 11 is parametrized and the corresponding optimal volume is evaluated using the *Interior-Point Method (IPM)* in MATLAB. A Pareto curve of optimal volume against the respec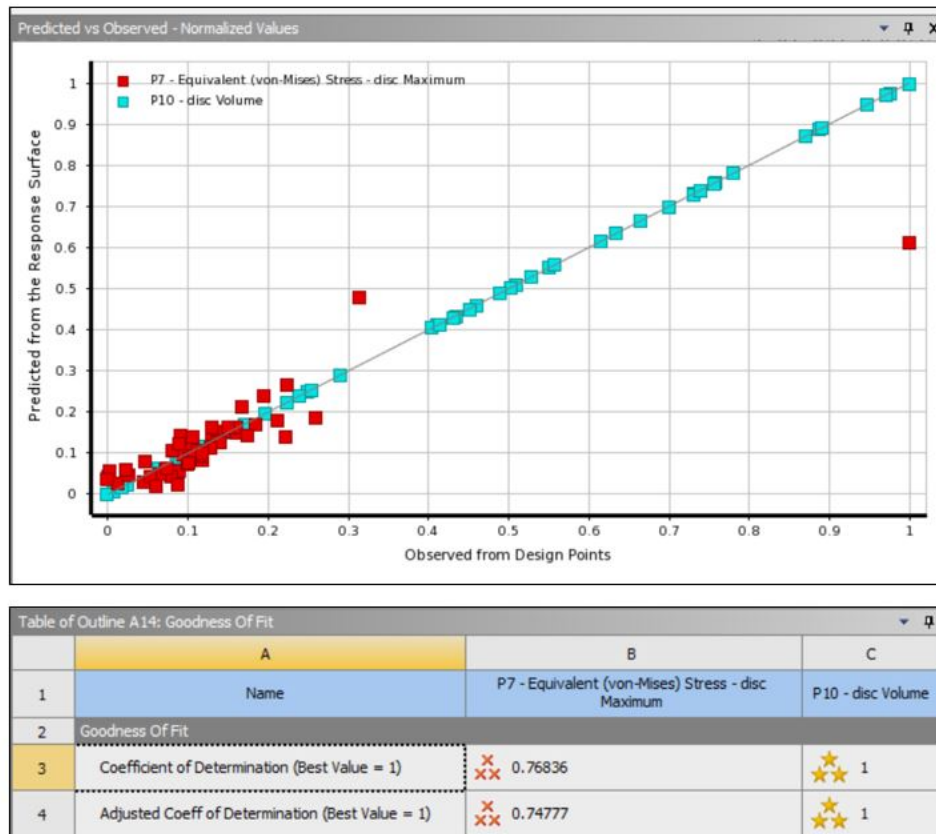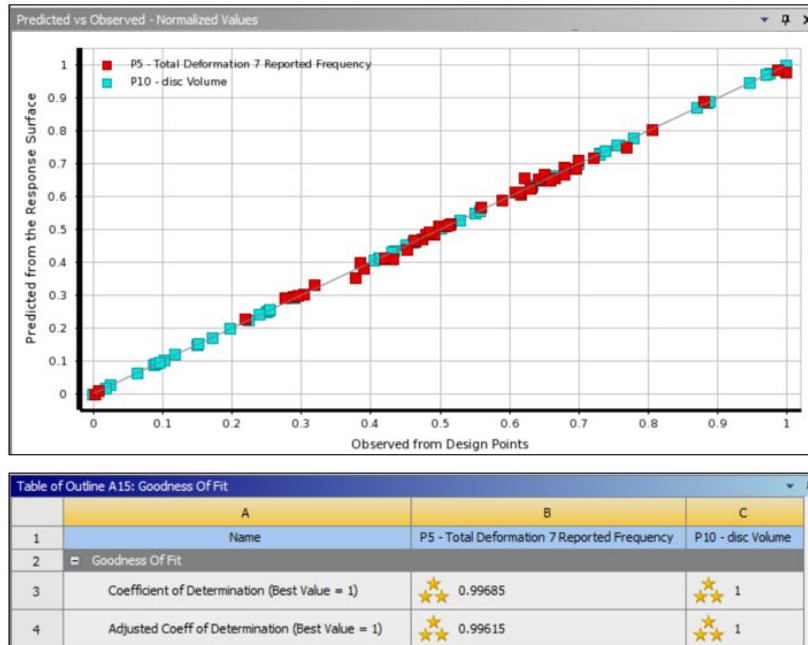tive stress upper bound is plotted using the IPM and MOGA algorithms as shown in Figure 13. All the MATLAB codes are attached in Appendix B. As observed from the figure, the curves obtained using both the algorithms are in good agreement with each other. Using the Pareto curve, one can easily obtain the optimal volume value for a given stress upper bound constraint.

Figure 13: Correlation of Pareto curves from IPM (fmincon) and MOGA for structural analysis

It is observed from the Lagrange multiplier values that constraints $g3, g5$ and $g7$ become active. The design variables optimal values obtained for 14 MPa stress upper bound are correlated with those obtained from ANSYS in the results section. The optimal design variables candidate points obtained in ANSYS by the implementation of MOGA are shown in Figure 14. The highlighted candidate point (least volume) is chosen as the best point which is used for validation purposes.

5.2   Modal Analysis Optimization

For the subsystem 2 optimization problem, the lower bound on the first natural frequency constraint from Equation 12 is parametrized and the corresponding optimal volume is evaluated using IPM in MATLAB. A Pareto curve of optimal volume against the respective frequency lower bound is plotted using the IPM and MOGA algorithms as shown in Figure 15. As observed from the figure, the curves obtained using both the algorithms are in good agreement

Figure 14: Optimal dimensions candidate points for structural analysis

with each other. Using the Pareto curve, one can easily obtain the optimal volume value for a given frequency lower bound constraint.

Figure 15: Correlation of Pareto curves from IPM (fmincon) and MOGA for modal analysis

It is observed from the Lagrange multiplier values that constraints $g3$ and $g8$ become active. The design variables optimal values obtained for 1200 Hz frequency lower bound are correlated with those obtained from ANSYS in the results section. The optimal design variables candidate points obtained in ANSYS by the implementation of MOGA are shown in Figure 16. The highlighted candidate point (least volume) is chosen as the best point which is used for validation purposes.

Figure 16: Optimal dimensions candidate points for modal analysis

5.3 Thermal Analysis Optimization

For the subsystem 3 optimization problem, the upper bound on the temperature constraint from Equation 13 is parametrized and the corresponding optimal volume is evaluated using the *Interior-Point Method (IPM)* in MATLAB. A Pareto curve of optimal volume against the respective temperature upper bound is plotted using the IPM and MOGA algorithms as shown in Figure 17. As observed from the figure, the curves obtained using both the algorithms are in good agreement with each other. Using the Pareto curve, one can easily obtain the optimal volume value for a given temperature upper bound constraint.
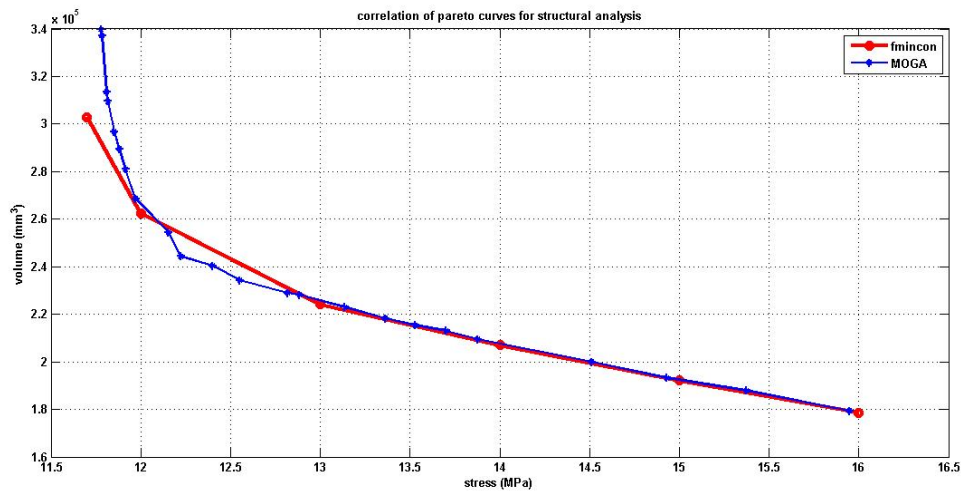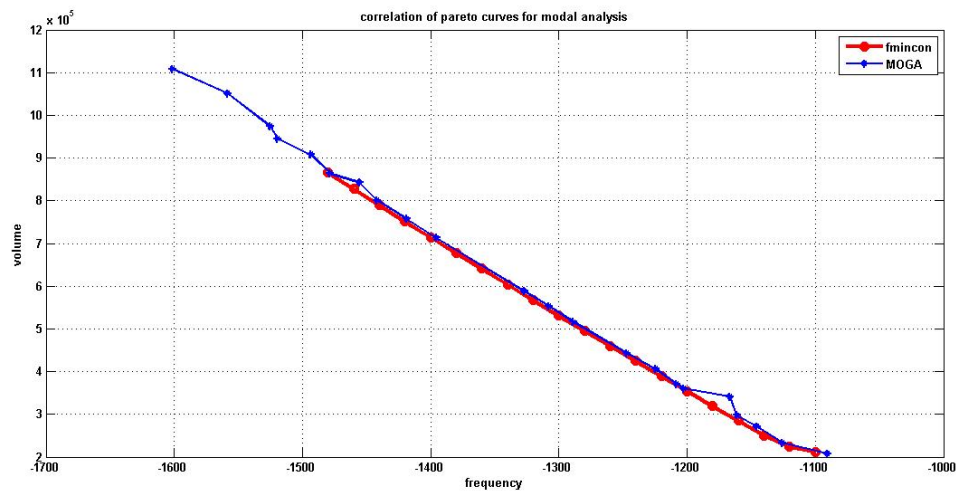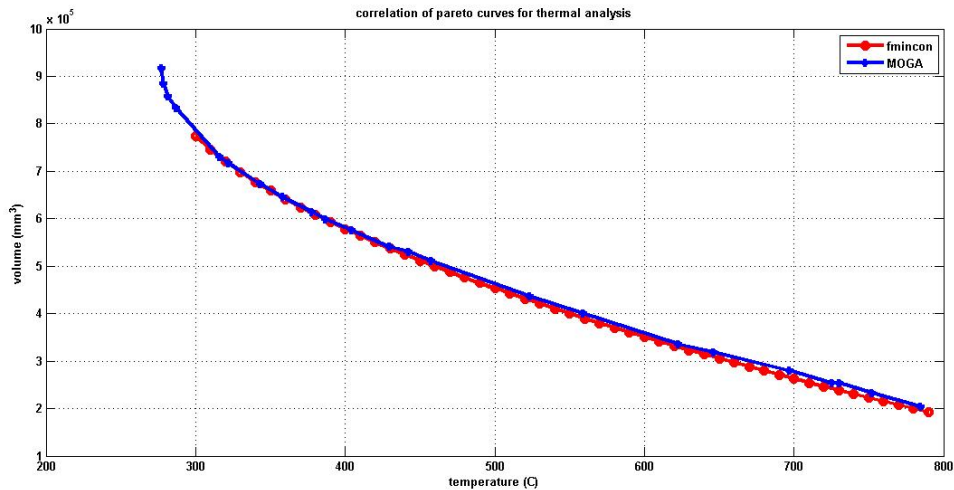

Figure 17: Correlation of Pareto curves from IPM (fmincon) and MOGA for thermal analysis

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It is observed from the Lagrange multiplier values that constraints $g3$ and $g9$ become active. The design variables optimal values obtained for 400^oC temperature upper bound are correlated with those obtained from ANSYS in the results section. The optimal design variables candidate points obtained in ANSYS by the implementation of MOGA are shown in Figure 18. The highlighted candidate point (least volume) is chosen as the best point which is used for validation purposes.

Figure 18: Optimal dimensions candidate points for thermal analysis

6 Discussion of Results

6.1 Structural Analysis Results

As observed from the Pareto curves in Figure 13, the optimal volume magnitude increases as the stress upper bound decreases. The objective here is to minimize the stress and volume which ensures a trade-off between the two. This trade-off makes the stress constraint ($g7$) active. The lower bound of the outer radius ($g3$) and thickness ($g5$) constraints are also active. This is confirmed by obtaining and analyzing the Langrange multipliers for all the constraints. Optimal values of the design variables for 14 MPa stress constraint are obtained from MATLAB (fmincon algorithm) and ANSYS (MOGA). Their correlation is shown in Table 2.

Parameter	Initial value	Optimal value from MATLAB (m)	Optimal value from ANSYS (a)	Error (%) $(a-m)/m$
P1	75.00	78.42	75.05	1.05
P2	125.00	124.00	124.19	0.15
P3	25.00	5.00	5.08	1.06
S	14.26	14.00	13.99	-0.07

Table 2: Correlation between MATLAB (IPM) and ANSYS (MOGA) optimal values for 14 MPa stress constraint

The error values in the table show that the MATLAB and ANSYS optimal values are in good agreement with each other. The stress constraint being active, the optimal stress value obtained from ANSYS should be 14 MPa. But as observed from the Table, the ANSYS value is 14.26 MPa. This has introduced errors in the disc dimension values. These errors will reduce if MOGA generates an optimal stress value closer to 14 MPa in ANSYS.

6.2   Modal Analysis Results

As observed from the Pareto curves in Figure 15, the optimal volume magnitude increases as the frequency lower bound increases. The objective here is to maximize the frequency and minimize the volume which ensures a trade-off between the two. This trade-off makes the frequency constraint ($g8$) active. The lower bound of the outer radius ($g3$) constraint is also active. This is confirmed by obtaining and analyzing the Langrange multipliers for all the constraints. Optimal values of the design variables for 1200 Hz frequency constraint are obtained from MATLAB (fmincon algorithm) and ANSYS (MOGA). Their correlation is shown in Table 3.

Parameter	Initial value	Optimal value from MATLAB (m)	Optimal value from ANSYS (a)	Error (%) $(a-m)/m$
P1	75.00	78.25	83.40	6.58
P2	125.00	124.00	124.19	0.15
P3	25.00	7.76	7.23	-6.82
F	1590.00	1200.00	1263.62	5.30

Table 3: Correlation between MATLAB (IPM) and ANSYS (MOGA) optimal values for 1200 Hz frequency constraint

The error values in the table show that the MATLAB and ANSYS optimal values are in good agreement with each other. The frequency constraint being active, the optimal frequency value obtained from ANSYS should be 1200 Hz. But as observed from the Table, the ANSYS value is 1263.62 Hz. This has introduced errors in the disc dimension values. These errors will reduce if MOGA generates an optimal frequency value close to 1200 Hz in ANSYS.

6.3   Thermal Analysis Results

As observed from the Pareto curves in Figure 17, the optimal volume magnitude increases as the temperature upper bound decreases. The objective here is to minimize the temperature and volume which ensures a trade-off between the two. This trade-off makes the temperature constraint ($g9$) active. The lower bound of the outer radius ($g3$) constraint is also active. This is confirmed by obtaining and analyzing the Langrange multipliers for all the constraints. Optimal values of the design variables for 400^oC temperature constraint are obtained from MATLAB (fmincon algorithm) and ANSYS (MOGA). Their correlation is shown in Table 4.

Parameter	Initial value	Optimal value from MATLAB (m)	Optimal value from ANSYS (a)	Error (%) $(a-m)/m$
P1	75.00	83.16	82.77	-0.46
P2	125.00	124.00	124.59	0.47
P3	25.00	13.69	15.54	13.51
T	321.70	400.00	385.81	-3.54

Table 4: Correlation between MATLAB (IPM) and ANSYS (MOGA) optimal values for 400^oC temperature constraint

The error values in the table show that the MATLAB and ANSYS optimal values are in good agreement with each other. The temperature constraint being active, the optimal temperature value obtained from ANSYS should be 400^oC. But as observed from the Table, the ANSYS value is 321.7^oC. This has introduced errors in the disc dimension values. These errors will reduce if MOGA generates an optimal stress value closer to 400^oC in ANSYS.

7 System Integration Study

After performing the individual subsystem level optimization, an attempt is made to integrate all the subsystems. There are four objective functions in this problem. As it is not possible to plot 4D Pareto curves, three objective functions are considered at a time so that 3D Pareto curves can be easily plotted from them in MATLAB. This study helps us understand the relationship between three objective functions at a time.

7.1 Combined Modal & Thermal Optimization

The mathematical model for this system level optimization is as follows:

$Min : f1, -f3, f4$

s.t.
$g1, g2, g3, g4, g5, g6, g8$ and $g9$

Optimization is performed in MATLAB using the single objective IPM (fmincon) method. This algorithm being used for single objective optimization problem, the volume function is considered to be the primary objective and the frequency and temperature objective functions are considered as constraints. The lower and upper bounds on frequency and temperature respectively are varied and a Pareto curve for volume, frequency and temperature is obtained as shown in Figure 19. The optimal volume value for 1200 Hz lower bound on frequency and 400^oC upper bound on temperature is obtained from this Pareto curve and is correlated with a MOGA value obtained from ANSYS.

Figure 19: Pareto curve for volume, frequency and temperature

As observed from this Pareto curve, for a given temperature constraint, the optimal volume does not vary with the frequency constraint. Thus, we have the temperature constraint ($g9$) active. This is verified from the Lagrange multipliers obtained from MATLAB. MATLAB (fmincon algorithm) and ANSYS (MOGA) correlation for one of the temperature and frequency constraints is presented in Table 5. The candidate points obtained from ANSYS are shown in Figure 20. The highlighted point (least volume) is chosen as the best optimal point.

Figure 20: Optimal dimensions candidate points for combined modal and thermal analysis

Parameter	Initial value	Optimal value from MATLAB (m)	Optimal value from ANSYS (a)	Error (%) $(a-m)/m$
P1	75.00	83.16	88.19	6.04
P2	125.00	124.00	124.10	0.08
P3	25.00	13.69	14.64	6.93
F	1590.00	1310.60	1220.20	-6.89
T	321.70	400.00	399.80	-0.05

Table 5: Correlation between MATLAB and ANSYS optimal values for 1200 Hz frequency constraint and $400^o C$ temperature constraint

As observed from the Table 5, there is no significant difference in the error percentage between ANSYS and MATLAB values. Moreover, MATLAB gives a better optimal point (lower dimensions and volume) as compared to the ANSYS value. MOGA solutions do not satisfy the KKT conditions. The initial volume being $9.96 * 10^5 mm^3$, MATLAB optimal solution gives the minimum volume to be $5.78 * 10^5 mm^3$ i.e. an improvement in the volume reduction of **41.9%**.

7.2   Combined Modal & Structural Optimization

The mathematical model for this system level optimization is as follows:

$Min : f1, f2, -f3$

s.t.
$g1, g2, g3, g4, g5, g6, g8$ and $g7$

Optimization is performed in MATLAB using the single objective IPM (fmincon) method. This algorithm being used for single objective optimization problem, the volume function is considered to be the primary objective and the frequency and stress objective functions are considered as constraints. The lower and upper bounds on frequency and stress respectively are varied and a Pareto curve for volume, frequency and stress is obtained as shown in Figure 21. The optimal volume value for 1200 Hz lower bound on frequency and 14 MPa upper bound on stress is obtained from this Pareto cu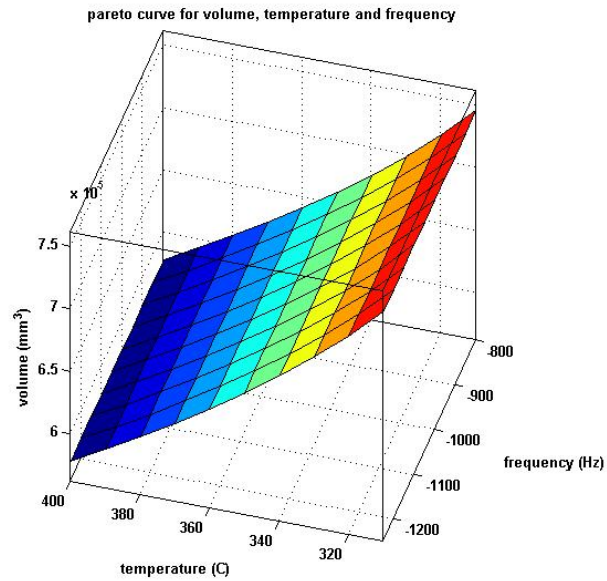rve and is correlated with a MOGA value obtained from ANSYS. As observed from this Pareto curve, for a given frequency constraint, the optimal volume doesn't vary with the stress constraint. Thus, we have the frequency constraint ($g8$) active. This is verified from the Lagrange multipliers obtained from MATLAB. MATLAB (fmincon algorithm) and ANSYS (MOGA) correlation for one of the stress and frequency constraints is presented in Table 6. The candidate points obtained from ANSYS are shown in Figure 22. The highlighted point (least volume) is chosen as the best optimal point.
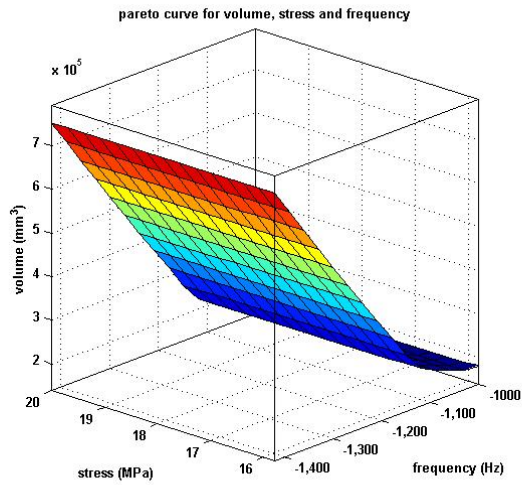
Figure 21: Pareto curve for volume, frequency and stress

Figure 22: Optimal dimensions candidate points for combined modal and structural analysis

Parameter	Initial value	Optimal value from MATLAB (m)	Optimal value from ANSYS (a)	Error (%) $(a-m)/m$
P1	75.00	78.25	81.64	4.33
P2	125.00	124.00	124.33	0.26
P3	25.00	7.76	6.61	-14.81
F	1590.00	1200	1261.30	5.10
S	14.26	12.77	13.20	3.36

Table 6: Correlation between MATLAB and ANSYS optimal values for 1200 Hz frequency constraint and 14 MPa stress constraint

As observed from the Table 6, there is no significant difference in the error percentage between ANSYS and MATLAB values. Moreover, MATLAB gives a better optimal point (lower dimensions and volume) as compared to the ANSYS value. The initial volume being $9.96 * 10^5 mm^3$, MATLAB optimal solution gives the minimum volume of $3.54 * 10^5 mm^3$ i.e. an improvement in the volume reduction of **64.4%**.

7.3 Combined Structural, Modal & Thermal Optimization

The mathematical model for this system optimization is as follows:

$Min : f1, f2, -f3, f4$

s.t.
$g1, g2, g3, g4, g5, g6$

This combined system level optimization is performed in MATLAB and ANSYS using the MOGA algorithm. The optimization procedure is as shown in Figure 23.
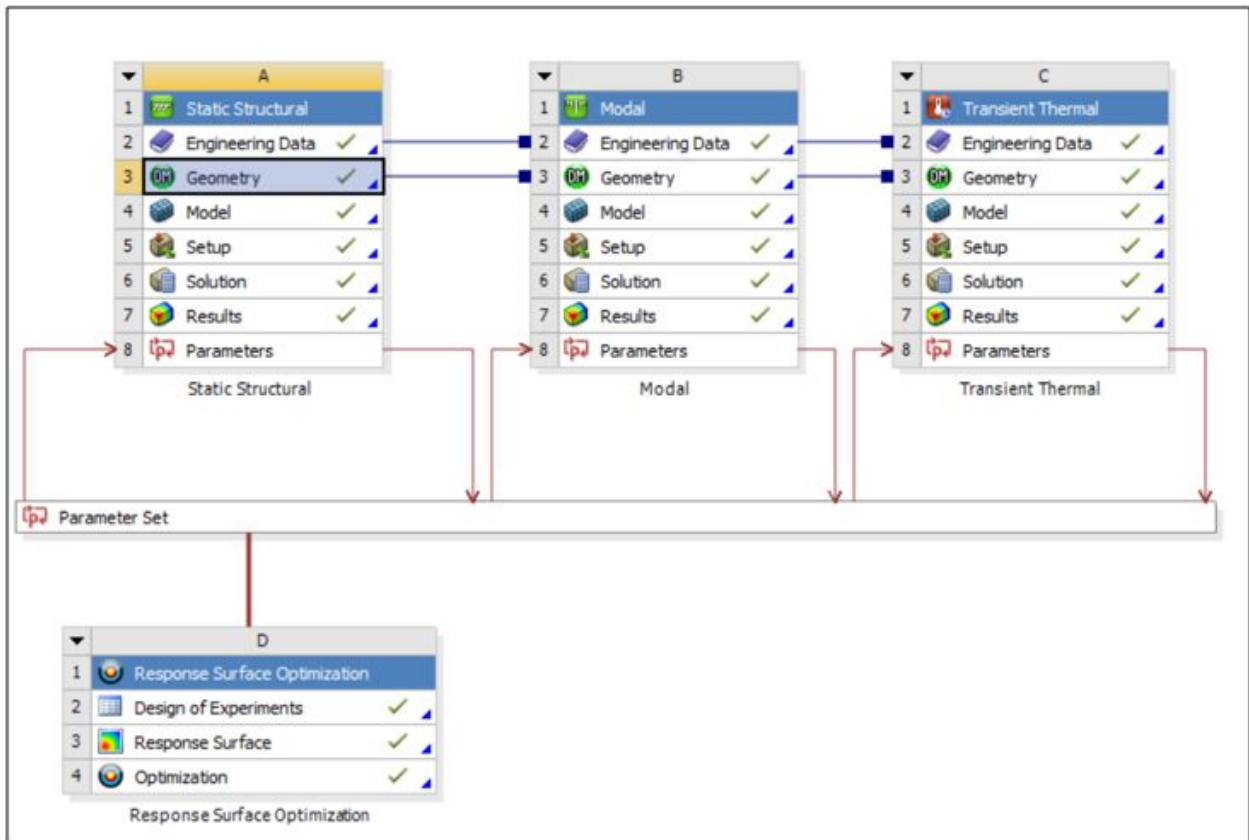
Figure 23: System optimization procedure

Corresponding points chosen from MATLAB and ANSYS are validated in Table 7. The highlighted point (least volume) from Figure 24 is chosen as the best optimal point.

Figure 24: Optimal dimensions candidate points for combined structural, modal and thermal analysis

Parameter	Initial value	Optimal value from MATLAB (m)	Optimal value from ANSYS (a)	Error (%) $(a-m)/m$
P1	75.00	76.11	81.48	7.05
P2	125.00	124.83	124.33	-0.40
P3	25.00	14.34	13.80	-3.76
F	1590.00	1340.60	1328.1	-0.93
S	14.26	13.28	13.35	0.53
T	321.70	370.75	414.40	11.77
V	$9.96*10^5$	$6.45*10^5$	$6.17*10^5$	-4.34

Table 7: Correlation between MATLAB and ANSYS optimal values for combined structural, modal and thermal analysis using MOGA

As observed from Table 7, the initial volume being $9.96*10^5 mm^3$, MATLAB optimal solution gives the minimum volume as $6.45*10^5 mm^3$ i.e. an improvement in the volume reduction of **35.2%**. The final optimal dimensions (P1, P2 and P3) as given by the MOGA algorithm in MATLAB are 76.11 mm, 124.83 mm and 14.34 mm respectively.

Initial Geometry

Volume reduction 35.2%
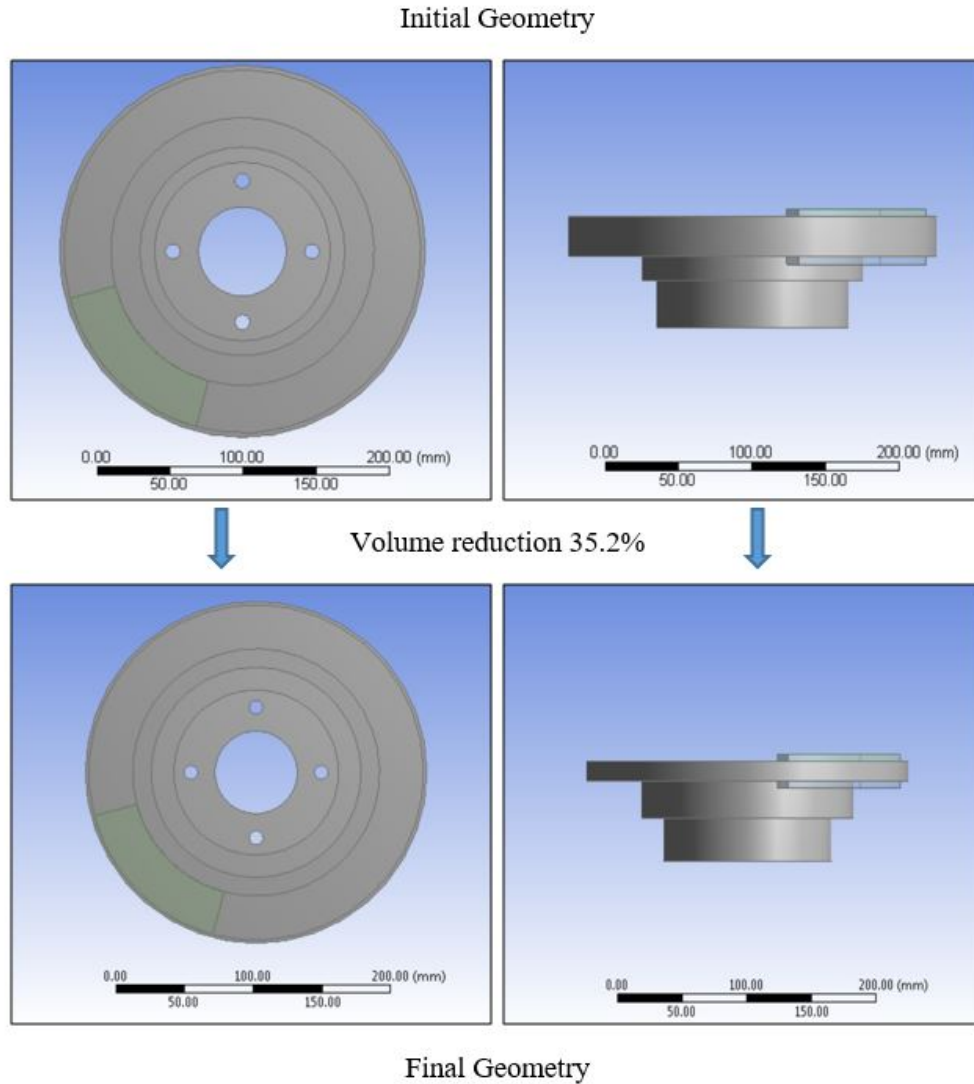
Final Geometry

Figure 25: Initial and final brake disc geometries

8   References

[1] P. Y. Papalambros, Principles of Optimal Design: Modeling and Computation, 2016.

[2] M. Reibenschuh et. al, Modelling and Analysis of Thermal and Stress Loads in Train Disc Brakes Braking from 250 km/h to Standstill, *Journal of Mechanical Engineering*, 2009.

[3] D. C. Montgomery, Design and Analysis of Experiments, John Wiley & Sons, Inc., Eighth Edition.

[4] G. M. Nathy et. al, Coupled Structural / Thermal Analysis of Disc Brake, *IJRET*.

[5] ANSYS and MATLAB manual.

[6] M. Reibenschuh and F. Cus, Stress Analysis Of A Brake Disc Considering Centrifugal Load, *Journal of Production Engineering*, Vol. 12.

[7] Y. Ren, MAE 598 - Design Optimization Notes, 2016.

9 Appendix

9.1 A

The DOE points obtained using the LHS method are as follows:

Table of Schematic D2: Design of Experiments (Latin Hypercube Sampling Design : User-Defined Samples : Random Generator Seed = 0 : Number of Samples = 50)

	Name	Update Order	P1 - rotor_ID (mm)	P2 - rotor_OD (mm)	P3 - rotor_thickness (mm)	P5 - Total Deformation 7 Reported Frequency (Hz)	P7 - Equivalent (von -Mises) Stress - disc Maximum (MPa)	P8 - Temperature Maximum (C)	P10 - disc Volume (mm^3)
1	A	B	C	D	E	F	G	H	I
2	1	19	74.88	125.82	26.34	1599.4	14.971	317.69	1.0569E+06
3	2	28	79.2	140.9	14.02	1057.6	14.753	403.5	8.242E+05
4	3	2	66.72	146.1	6.98	589.7	14.291	683.79	5.5418E+05
5	4	23	76.8	139.34	25.9	1317.6	15.86	316.67	1.3174E+06
6	5	4	67.68	129.98	8.74	977.32	12.838	566.87	5.249E+05
7	6	47	88.32	132.06	18.42	1153.2	15.226	350.89	8.1791E+05
8	7	22	76.32	142.98	25.02	1245.6	16.296	318.72	1.3649E+06
9	8	46	87.84	139.86	5.22	878.91	15.07	884.74	4.5248E+05
10	9	10	70.56	133.62	11.82	1084.5	12.877	450.87	6.7436E+05
11	10	7	69.12	149.74	7.42	594.41	14.459	648.77	6.028E+05
12	11	14	72.48	124.26	8.3	1211.8	13.034	602.67	4.683E+05
13	12	6	68.64	136.74	14.9	1109.3	13.597	389.39	8.4463E+05
14	13	18	74.4	135.7	10.94	1046.4	13.266	476.28	6.5185E+05
15	14	31	80.64	148.18	13.14	909.45	15.457	420.08	8.6923E+05
16	15	13	72	128.94	26.78	1588	15.457	315.37	1.1636E+06
17	16	40	84.96	144.54	23.26	1107.2	16.486	324	1.246E+06
18	17	38	84	137.26	9.18	1024.9	14.003	544.26	5.835E+05
19	18	27	78.72	148.7	22.38	1092.6	15.575	327.25	1.3433E+06
20	19	5	68.16	134.66	17.54	1237.4	14.39	358.08	9.3158E+05
21	20	3	67.2	149.22	19.3	1012.4	14.325	343.82	1.2616E+06
22	21	17	73.92	140.38	15.78	1100.7	14.502	377.31	9.1361E+05
23	22	41	85.44	128.42	22.82	1293.1	15.121	326.23	9.0803E+05
24	23	48	88.8	146.62	20.62	1013.8	17.975	336.15	1.1437E+06
25	24	8	69.6	127.38	13.58	1295.9	13.194	413.13	6.786E+05
26	25	34	82.08	137.78	7.86	969.92	14.123	617.85	5.389E+05
27	26	26	78.24	134.14	17.1	1246.4	14.483	362.37	8.604E+05
28	27	37	83.52	133.1	21.06	1247.1	14.936	333.37	9.524E+05
29	28	30	80.16	127.9	11.38	1261.9	13.581	464.5	5.8463E+05

Figure 26: LHS DOE points

Table of Schematic D2: Design of Experiments (Latin Hypercube Sampling Design : User-Defined Samples : Random Generator Seed = 0 : Number of Samples = 50)

	Name	Update Order	P1 - rotor_ID (mm)	P2 - rotor_OD (mm)	P3 - rotor_thickness (mm)	P5 - Total Deformation 7 Reported Frequency (Hz)	P7 - Equivalent (von -Mises) Stress - disc Maximum (MPa)	P8 - Temperature Maximum (C)	P10 - disc Volume (mm^3)
1	A	B	C	D	E	F	G	H	I
28	27	37	83.52	133.1	21.06	1247.1	14.936	333.37	9.524E+05
29	28	30	80.16	127.9	11.38	1261.9	13.581	464.5	5.8463E+05
30	29	49	89.28	132.58	6.54	1068.7	14.166	722.59	4.612E+05
31	30	15	72.96	144.02	16.66	1055.3	14.772	366.94	1.0113E+06
32	31	35	82.56	145.58	10.5	894.46	14.544	490.84	7.1256E+05
33	32	39	84.48	143.5	6.1	808	14.603	769.64	5.033E+05
34	33	20	75.36	135.18	18.86	1276	14.769	347.01	9.5875E+05
35	34	42	85.92	142.46	21.94	1106.5	16.025	329.23	1.1409E+06
36	35	16	73.44	145.06	17.98	1073.1	14.432	354.08	1.0899E+06
37	36	11	71.04	126.34	9.62	1184.5	12.821	528.56	5.2771E+05
38	37	45	87.36	138.3	16.22	1078.1	14.972	372.14	8.4214E+05
39	38	44	86.88	130.5	21.5	1228.2	15.677	331.53	8.9501E+05
40	39	25	77.76	124.78	15.34	1404.3	13.911	388.21	6.7988E+05
41	40	50	89.76	147.66	19.74	982.6	29.205	341.69	1.1182E+06
42	41	24	77.28	131.02	12.7	1226.2	13.7	429.75	6.6586E+05
43	42	21	75.84	147.14	12.26	886.46	14.753	439.67	8.2653E+05
44	43	32	81.12	126.86	10.06	1255.6	13.831	509.83	5.3366E+05
45	44	9	70.08	131.54	23.7	1480.4	14.575	322.59	1.1173E+06
46	45	36	83.04	129.46	25.46	1366	16.47	318.01	1.0291E+06
47	46	33	81.6	141.42	24.58	1204.2	17.071	319.87	1.265E+06
48	47	43	86.4	125.3	14.46	1216.6	14.028	400.45	6.268E+05
49	48	29	79.68	136.22	5.66	867.33	14.264	823.56	4.4485E+05
50	49	12	71.52	141.94	24.14	1275.6	15.3	321.11	1.339E+06
51	50	1	66.24	138.82	20.18	1211.1	14.299	338.16	1.1258E+06

Figure 27: LHS DOE points (continued)

9.2 B

MOGA and fmincon (IPM) MATLAB codes for subsystem and combined system level optimization are attached below. Respective excel sheet containing all the DOE points should be provided for the codes to work. The subsystem level codes provided are only for the modal analysis. Changes can be easily made to make them work for structural and thermal analyses.

```matlab
%%ALL CODES ARE WRITTEN BY ADITYA VIPRADAS AND SHARAN KISHORE%%
%%(MEMBERS OF MAE598 DESIGN OPTIMIZATION TEAM 11)%%

%%%%%PLOT PARETO CURVES USING FMINCON AND MOGA%%%%%
function [xfmin, ffmin, vfmin, xmoga, fmoga, lambda] = corr_algos_modal ...
    (filename, fstar, N, string1, string2)
    clc;
    [xfmin, ffmin, vfmin, lambda] = fmincon_algo(filename, fstar, N);
    [xmoga, fmoga] = MOGA(filename);
    [fmoga_sort, a_order] = sort(fmoga(:,2));
    vmoga = fmoga(a_order,1);
    figure();
    plot(-ffmin, vfmin,'-o',fmoga_sort, vmoga,'-*');
    grid on;
    legend('fmincon','MOGA');
    xlabel(string2);
    ylabel('volume');
    str = strcat('correlation of pareto curves for ',string1,...
        ' analysis');
    title(str);
end

%%%%%EXTRACT DOE DATA FROM EXCEL FOR COMBINED SYSTEM OPTIMIZATION%%%%%
%%%%%PERFORM REGRESSION ANALYSIS ON THE DATA POINTS%%%%%
function [x, f, v, t, s, R2f, R2v, R2t, R2s] = extract(filename)
    global bv bf bt bs
    clc;
    %extract
    data = xlsread(filename);
    x = data(:,1:3);
    f = data(:,4);
    t = data(:,5);
    s = data(:,6);
    v = data(:,7);
    %quadratic fit
    X = [x,x.^2,ones(size(x,1),1)]

    %freq regression
    bf = (X'*X)\X'*f;

    %vol regression
    bv = (X'*X)\X'*v;

    %temp regression
    bt = (X'*X)\X'*t;

    %stress regression
    bs = (X'*X)\X'*s;

    rf = X*bf-f;
    %R2 value for freq
    R2f = 1-sum(rf.^2)/sum((f - mean(f)).^2);

    rv = X*bv-v;
    %R2 value for volume
```

```matlab
    R2v = 1-sum(rv.^2)/sum((v - mean(v)).^2);

    rt = X*bt-t;
    %R2 value for temp
    R2t = 1-sum(rt.^2)/sum((t - mean(t)).^2);

    rs = X*bs-s;
    %R2 value for temp
    R2s = 1-sum(rs.^2)/sum((s - mean(s)).^2);
end

%%%%%EXTRACT DOE DATA FROM EXCEL FOR SUBSYSTEM OPTIMIZATION%%%%%
function [x, f, v, R2f, R2v] = extract(filename)
    global bv bf
    clc;
    %extract
    data = xlsread(filename);
    x = data(:,1:3);
    f = data(:,4);
    v = data(:,7);
    %quadratic fit
    X = [x,x.^2,ones(size(x,1),1)]

    %freq regression
    bf = (X'*X)\X'*f;

    %vol regression
    bv = (X'*X)\X'*v;

    rf = X*bf-f;
    %R2 value for freq
    R2f = 1-sum(rf.^2)/sum((f - mean(f)).^2);

    rv = X*bv-v;
    %R2 value for volume
    R2v = 1-sum(rv.^2)/sum((v - mean(v)).^2);
end

%%%%%FMINCON IMPLEMENTATION FOR SYSTEM OPTIMIZATION%%%%%
function [xval, lambda] = fmincon_algo(filename, fcon, tcon, scon)
    %nf = 2; % number of objective functions
    %N = 50; % number of points for plotting
    global bf bv bt bs fcon tcon scon

    [x, f, v, t, s, R2f, R2v, R2t, R2s] = extract(filename);

    %objective function
    obj = @fmincon_obj

    %Define the upper and lower bounds
    lb = [66 124 5];
    ub = [90 150 27];

    %Define the equality constraints
    Aeq = [];
```

```matlab
    beq = [];

    %Define the inequality constraints
    A = [];
    b = [];

    h0 = [70 130 20];

    options.Algorithm = 'interior-point';
    %nonlinear constraints
    nonlcon = @quadcon;
    [h,fval,exitflag,output,lambda,grad,hessian] = ...
        fmincon(obj, h0, A, b, Aeq, beq, lb, ub, nonlcon, options);
    %disp(xval{2});
    %disp(freq);
    %plot(-freq, vol);
    %grid on;
end

%%%%%FMINCON IMPLEMENTATION FOR SUBSYSTEM OPTIMIZATION%%%%%
function [xval, freq, vol, lambda] = fmincon_algo(filename, fstar, N)
    %nf = 2; % number of objective functions
    %N = 50; % number of points for plotting
    global bf bv fstart

    fstart = fstar;

    [x, f, v, R2f, R2v] = extract(filename);

    %objective function
    obj = @fmincon_obj

    %Define the upper and lower bounds
    lb = [66 124 5];
    ub = [90 150 27];

    %Define the equality constraints
    Aeq = [];
    beq = [];

    %Define the inequality constraints
    A = [];
    b = [];

    h0 = [50 50 20];
    freq = [];
    vol = [];

    for i=1:N
        %nonlinear constraints
        nonlcon = @quadcon;
        [h,fval,exitflag,output,lambda,grad,hessian] = ...
            fmincon(obj, h0, A, b, Aeq, beq, lb, ub, nonlcon);
        xval{i} = h;
        freq(i) = fmincon_con(h);
```

```matlab
        vol(i) = fmincon_obj(h);
        fstart = fstart + 20;
    end
    %disp(xval{2});
    %disp(freq);
    %plot(-freq, vol);
    %grid on;
end

%%%%%DEFINE THE NONLINEAR CONSTRAINTS FOR SUBSYSTEM%%%%%
function [c,ceq] = quadcon(h)
    global bf fstart
    c =  -1*fmincon_con(h) + fstart;
    ceq = [];
end

%%%%%DEFINE THE NONLINEAR CONSTRAINTS FOR ENTIRE SYSTEM%%%%%
function [c,ceq] = quadcon(h)
    global bf bt fcon tcon scon
    c(1) = -1*fmincon_conf(h) + fcon;
    c(2) = fmincon_cont(h) - tcon;
    c(3) = fmincon_cons(h) - scon;
    ceq = [];
end

%%%%%DEFINE THE OBJECTIVE FUNCTION%%%%%
function y1 = fmincon_obj(x)
    global bv
    y1 = 0;
    X = [x, x.^2, ones(1)];
    for i = 1:length(X)
        y1 = y1 + bv(i)*X(i);
    end
end

%%%%%DEFINE THE NONLINEAR CONSTRAINTS%%%%%
function c1 = fmincon_con(x)
    global bf
    c1 = 0;
    X = [x, x.^2, ones(1)];
    for i = 1:length(X)
        c1 = c1 + bf(i)*X(i);
    end
end

%%%%%DEFINE THE SUBSYSTEM OBJECTIVE FOR MOGA%%%%%
function y = MOGA_obj(x)
    global bf bv;
    y = zeros(2,1);
    %[x, bf, bv, f, v, R2f, R2v] = extract(filename);
    X = [x, x.^2, ones(1)];
    for i = 1:length(X)
        y(1) = y(1) + bv(i)*X(i);
        y(2) = y(2) + bf(i)*X(i);
    end
```

```matlab
        y(2) = -y(2);
end


%%%%IMPLEMENT SYSTEM LEVEL MOGA%%%%
function [xmoga, fmoga] = corr_algos_modal_thermal_structural(filename)
    clc;
    %[xfmin, lambda] = fmincon_algo(filename, fcon, tcon, scon);
    [xmoga, fmoga] = MOGA(filename);
end


%%%%DEFINE SYSTEM LEVEL MOGA OBJECTIVE%%%%
function y = MOGA_obj(x)
    global bf bv bt bs;
    y = zeros(4,1);
    %[x, bf, bv, f, v, R2f, R2v] = extract(filename);
    X = [x, x.^2, ones(1)];
    for i = 1:length(X)
        y(1) = y(1) + bv(i)*X(i);
        y(2) = y(2) + bf(i)*X(i);
        y(3) = y(3) + bt(i)*X(i);
        y(4) = y(4) + bs(i)*X(i);
    end
    y(2) = -y(2);
end


%%%%IMPLEMENT THE MOGA ALGORITHM%%%%
function [xval, fval] = MOGA(filename)
    global bf bv bt bs
    [x, f, v, t, s, R2f, R2v, R2t, R2s] = extract(filename);
    FitnessFunction = @MOGA_obj; % Function handle to the fitness function
    numberOfVariables = 3; % Number of decision variables
    lb = [66 124 5]; % Lower bound
    ub = [90 150 27]; % Upper bound
    A = []; % No linear inequality constraints
    b = []; % No linear inequality constraints
    Aeq = []; % No linear equality constraints
    beq = []; % No linear equality constraints
    options.PlotFcn = @gaplotpareto;
    options.TolFun = 1e-5;
    options.ParetoFraction = 0.5;
    %options.Generations = 1000;
    %options = optimset(@gamultiobj,'PlotFcn',@gaplotpareto);
    [xval,fval,exitFlag,Output] = gamultiobj(FitnessFunction, ...
        numberOfVariables,A,b,Aeq,beq,lb,ub,options);
    fprintf('The number of points on the Pareto front was: %d\n', ...
        size(x,1));
    fprintf('The number of generations was : %d\n', Output.generations);
    %plot(freq, vol);
    %grid on;
end
```