OPTIMISATION OF FUEL IN HYBRID ELECTRIC VEHICLE

PROJECT REPORT

Submitted in the partial fulfillment of the course

MAE 598 Design Optimisation

by

Mohammad Alzorgan

Dheeraj Kumar Madisetty

Submitted on

May 5th 2016

Table of contents

Tables in the report

Figures in the report

15	**Graph of SOC versus stages**
16	**Infeasibility graph**
17	**Torque comparison between engine and motor**
18	**Fuel consumption comparison between DP and rule based model.**

1. Abstract

Hybrid electric vehicles using regenerative braking are helping in reducing the fuel consumption and thereby decreasing the amount of pollution. It is powered by Internal Combustion Engine (ICE) which uses fuel in the chemical form and Electric Motor (EM) which uses the electrical energy stored in the storage devices like capacitor, battery etc. So any decision on the mode of fuel usage during the travel, arrived from a logical approach would be of great use in using the fuel efficiently. Because of regenerative braking acceleration and deceleration is done by using the reversible electrical path. As there are losses in the electrical path, the ICE will be compensating for the losses and making more use of electrical motor is beneficial because of its higher efficiency than the IC engine.

Parallel configuration of HEV is considered in our project. This enables the vehicle to run either on internal combustion engine or the electric motor or both. As the motor and the IC engine are both connected to the mechanical transmission any of the above mentioned three modes of powering is achieved.

The project aims at reduction of fuel consumption of a parallel configured hybrid electric vehicle by taking into account the terrain between the desired initial and final points of travel. So finding the best mode of fuel to be used, by considering the predictive terrain and using the torque split ratio between the motor and the engine as the indicator becomes an optimisation problem in itself.

usually optimization, but "s" is also ok.

2. Methodology

Dynamic programming is to be used for this purpose. Dynamic programming (DP) involves an optimisation problem in a backtracking approach. It divides the bigger problem into smaller sub-divisions and finds the optimal solution for the smaller subsystems. Once all the subsystems are solved for optimality, integration of them will give the optimal solution for the larger problem.

In this approach total journey is divided into equal number of sub divisions. At each subdivision the fuel consumption by the IC engine has to be minimized. We start backtracking from the destination point. The minimal fuel path to the previous point is obtained from all the possible paths and is reserved or stored as the minimal fuel path. This approach is carried backwards until the initial point is reached. Now all the minimal fuel paths starting from the endpoint to the initial point are joined to give the overall path for optimal fuel consumption.

The dynamic programming divides the bigger problem into N stages on to the horizontal axis. On the vertical axis there is the control function, in this case power from the IC engine. The values of the control function on the vertical axis will be equally spaced from 0 to Maximum power capacity of the IC engine. All the possible connections between the nodes of two consecutive stages are given. For this, it is important that the power from the engine or the motor can be changed from zero to any other possible value or vice versa within the time equal to the difference between the two stages.

The DP approach stage diagram with four stages is shown in the below figure. The project uses similar approach with more number of stages. In between each stage the energy consumption from the engine is calculated by taking an average of the engine power at the two stages and multiplying it with the time interval of the stage. So this is the cost for each subdivision that is to be minimized.

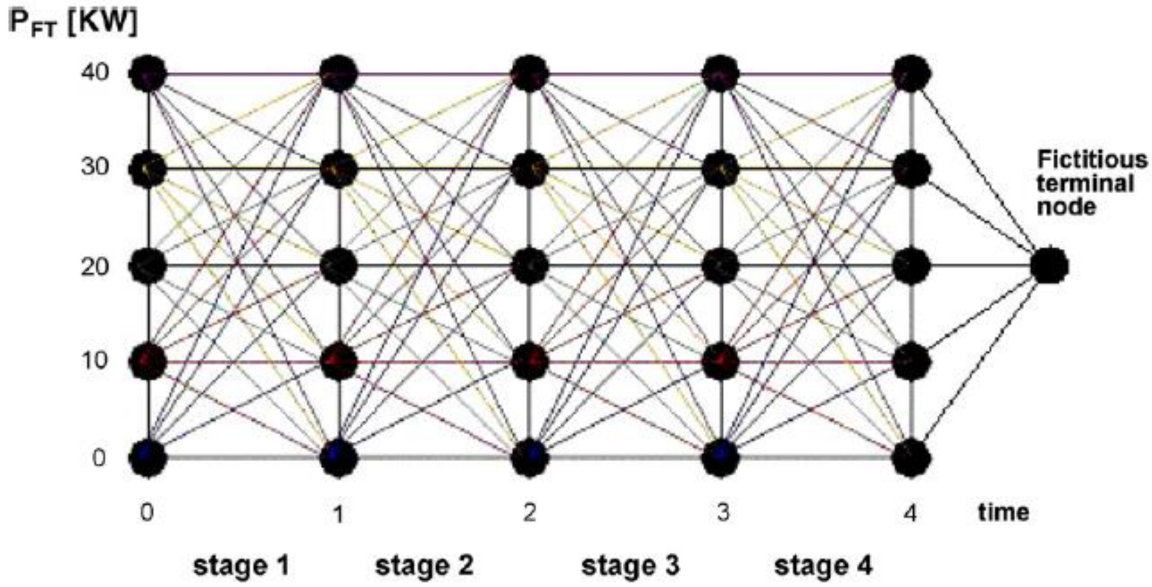


Figure 1

Dynamic programming diagram for a 4 stage problem

3. Nomenclature:

Table 1

$P_{ICE}(t)$	Power of the IC engine at time t (KW)
$P_{EM}(t)$	Power of the electric motor/ machine at time t (KW)
$P_{req}(t)$	Total power required at time t (KW)
η_{ICE}	Efficiency of the engine
η_{EM}	Efficiency of the electric motor
SOC(t)	State of charge at time t
SOC_{min}	Minimum state of charge

SOC_{max}	Maximum state of charge
P_{ICEmax}	Maximum power that can be delivered by the engine (KW)
P_{EMmax}	Maximum power that can be delivered by the motor (KW)
$E_{ICE}(t)$	Net energy consumed by the engine at time t (KWh)
$T_{ICE}(t)$	Torque of IC engine at time t (N-M)
$T_{EM}(t)$	Torque of electric motor at time t(N-M)

4. **Variable index**

Below is an extended table that includes all of the variables necessary to complete this project.

Table 2. Variable Index

Variable	Description
ICE_T	Engine Torque [Nm]
ICE_w	Engine Speed [rad/s]
ICE_m_f_dot	Fuel Flow Rate [kg/s]
ICE_T_req	Engine Torque Request [Nm]
TRAN_T	Transmission Torque [Nm]
TRAN_gear	Transmission Gear
FUEL_V_f	Fuel Volumetric Flow Rate [m^3/s]
WH_F_T	Wheel Torque [Nm]
WH_F_w	Wheel Speed [rad/s]
VEH_v	Vehicle Speed [m/s]
VEH_x	Vehicle Position [m]
VEH_mpg_diesel	Fuel Economy for Diesel [mpg]
VEH_mpg_gas	Fuel Economy for Gasoline[mpg]

DRIV_v_desired	Desired Vehicle Speed [m/s]
t	Time[s]
DRIV_alpha	Accelerator Pedal Position
DRIV_beta	Brake Pedal Position
VEH_F_a	Aerodynamic Drag Force [N]
VEH_F_g	Grade Force [N]
VEH_F_r	Rolling Resistance Force [N]
VEH_F_t	Wheel Tractive Force [N]
BATT_SOC	Battery State-Of-Charge
BATT_i	Battery Current[A]
BATT_V_cc	Battery Closed Circuit Voltage [V]
BATT_V_oc	Battery Open Circuit Voltage [V]
EM_P_in	Regenerative Power [W]
EM_T	Electric Motor Torque [Nm]
EM_T_Req	Electric Motor Torque Request [Nm]
EM_w	Electric Motor Speed [rad/s]
EM_eff	Electric Motor Efficiency

5. Mathematical model

- Total power required is supplied by the cumulative power of engine and power of electric machine.

$$\mathbf{P_{ICE}(t) + P_{EM}(t) = P_{req}(t)}$$

- But in reality the efficiencies of IC Engine and Electric machine also acts upon engine and motor respectively thus giving rise to a new practical equation. As we know efficiency ranges from 0 to 1

$$0 <= \eta_{ICE} <= 1 \quad \text{and } 0 <= \eta_{EM} <= 1$$

$$\eta_{\mathbf{ICE}} * \mathbf{P_{ICE}(t)} \quad + \mathbf{P_{em}(t)} * \eta_{\mathbf{EM}} = \mathbf{P_{req}(t)}$$

- Fuel energy consumed by the IC Engine at a time t is found by calculating the following integral

$$E_{ICE}(t) = \int_0^t (P_{ICE}(t)/\eta_{ICE}) * dt$$

- Unlike the energy consumed by the IC engine the power of electrical motor can also be negative when the regenerative break is applied.
  When the power is supplied from the EM, the final power driving the vehicle will be lesser due to the practical losses by a factor of efficiency.
  In case of regenerative kinetic energy of the vehicle is not entirely converted into electrical energy but is reduced by the factor of efficiency. So the negative power of electrical machine indicates that it is being charged and amount of inward charge is less than change in kinetic energy of the vehicle
  Thus there arise 2 equations for power of the motor.

$$f(P_{em}) = \eta_{em} * P_{em}(t) \ldots\ldots if\ P_{em} < 0$$

$$f(p_{em}) = \frac{P_{em}}{\eta_{em}} \ldots\ldots\ldots\ldots\ldots\ldots if\ p_{em} > 0$$ ✓

- SOC is the state of charge of the electrical storage device used by the battery. By using the above two equations total energy utilised by electrical storage device is given by the following equation

$$net\ energy = \int_0^t f(P_{em})dt$$

- Power of engine can only vary between the minimum (which is zero) and its maximum capacity

$$0 \le P_{ICE}(t) \le P_{ICEmax} \quad for\ all\ t$$

- Power of motor can vary between its minimum value and maximum value

$$P_{EM}min \le P_{EM}(t) \le P_{em\,max} \quad for\ all\ t$$

- State of charge can also vary only between the prescribed minimum and the maximum value

$$SOC_{min} \le SOC(t) \le SOC_{max} \quad for\ all\ t$$

Objective function

$f = \int_0^T \frac{P_{ICE}(t)}{\eta_{ICE}}\, dt$, to be minimized

Subject to constraints

$$SOC(t) = f(P_{req}(t) - P_{ICE}(t))$$

SOC can be expressed as a function of difference between required power and IC engine power

$$f\left(P_{req}(t) - P_{ICE}(t)\right) = \frac{P_{req}(t) - P_{ICE}(t)}{\eta_{em}} \; if \; P_{ICE}(t) < P_{req}(t)$$

$$f\left(P_{req}(t) - P_{ICE}(t)\right) = \eta_{em} * \left(P_{req}(t) - P_{ICE}(t)\right) if \; P_{ICE}(t) > P_{req}(t)$$

$$SOC(0) = 0.65$$

$$0 \leq P_{ICE}(t) \leq P_{ICEmax} \;\; for \; all \; t \; belongs \; to \; [0 \; T]$$

$$P_{em\,min} \leq P_{req}(t) - P_{IC}(t) \leq P_{EM\,max} \;\; for \; all \; t \; belongs \; to \; [0 \; T]$$

$$SOC_{min} \leq SOC(t) \leq SOC_{max} \;\; for \; all \; t \; belongs \; to \; [0 \; T]$$

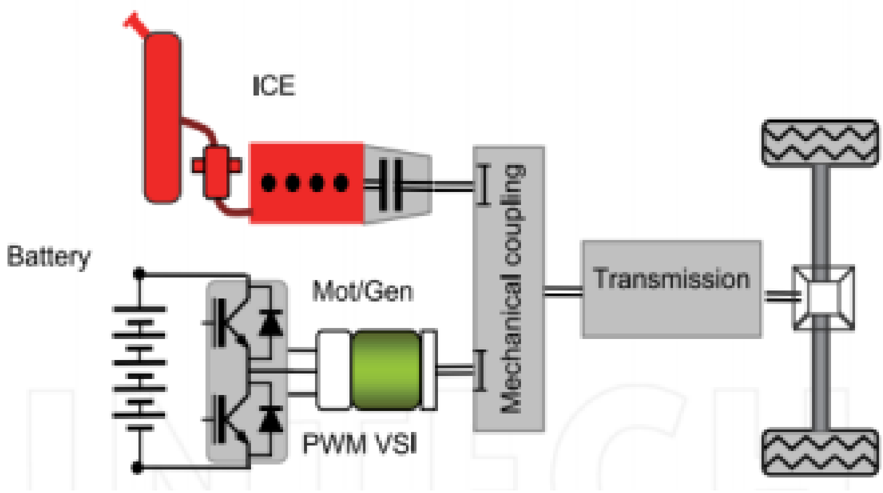
$$SOC_{min} = 0.4$$

$$SOC_{max} = 0.9$$

6. Model Analysis:

Modeling of parallel hybrid powertrain

This is a parallel hybrid vehicle configuration. Here, the front axle is driven by the internal combustion engine . The rear axle is driven by the electric machine. The forces from the front and rear wheels are summed to yield a total tractive force that is sent to the Vehicle block.



Component	Specs
Hybrid Battery Packs Power	30 kW
EM Power Max	50 kW @ 8000 rpm
ICE Power Max	100 kW @ 6000 rpm
ICE Max TRQ @ 6000 rpm	200 N.m
EM TRQ @ 8000 rpm	150 N.m

Figure 2

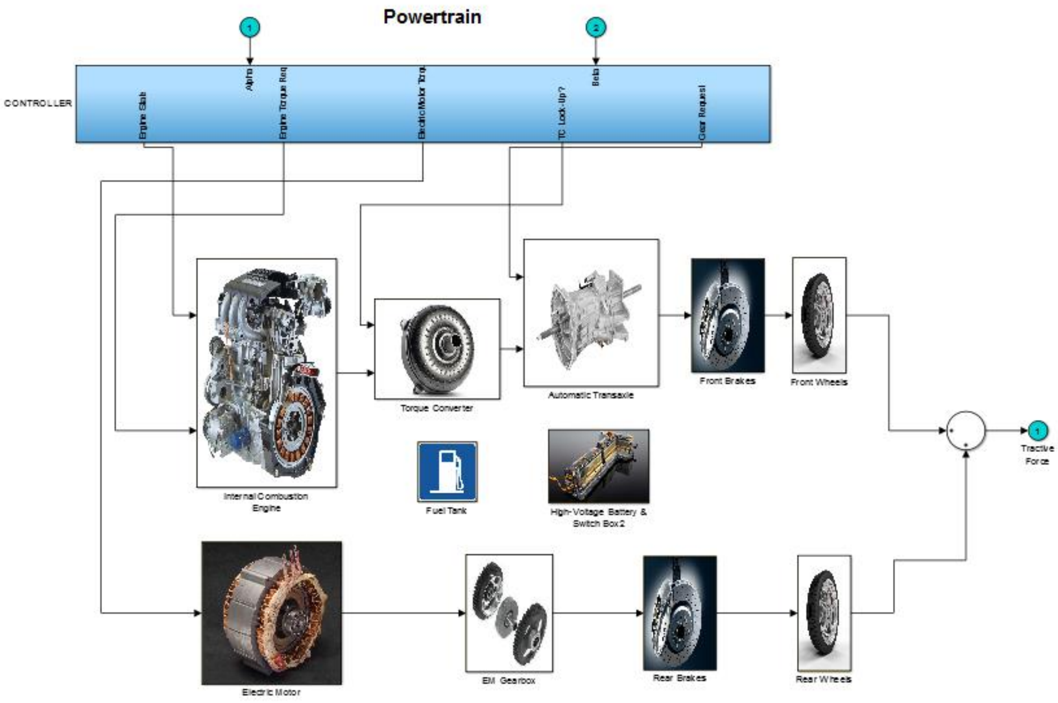


Figure 3

Symulink model of the power train

Modeling of Driver Model for drive cycle and speed mathching

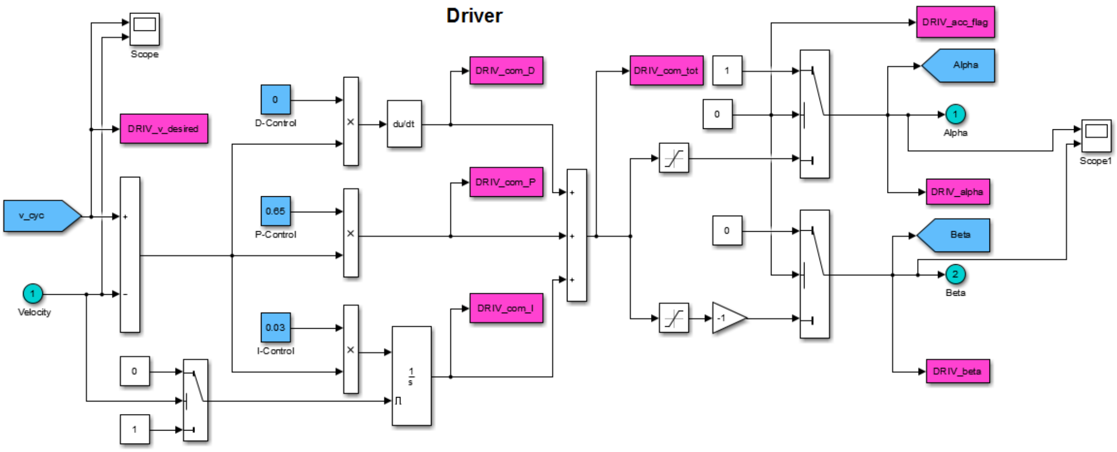


Figure 4

Modeling of Rule Based controller

Using an electric machine with a charge-sustaining strategy greatly increases the complexity of the controller. The reason for this is the torque request in the electric portion of the Powertrain is not the only concern as is the case with the internal combustion engine counterpart.

In order for the vehicle to be charge sustaining, the battery state-of-charge (SOC) must also be controlled. Therefore, the electric powertrain must be able to provide power by drawing energy from the battery, as well as, absorb power by putting energy back into the battery. Since the energy storage device is not an infinite well, there are limits to the quantity of power that can be drawn from, or given to, the unit. These constraints are built into the control strategy.

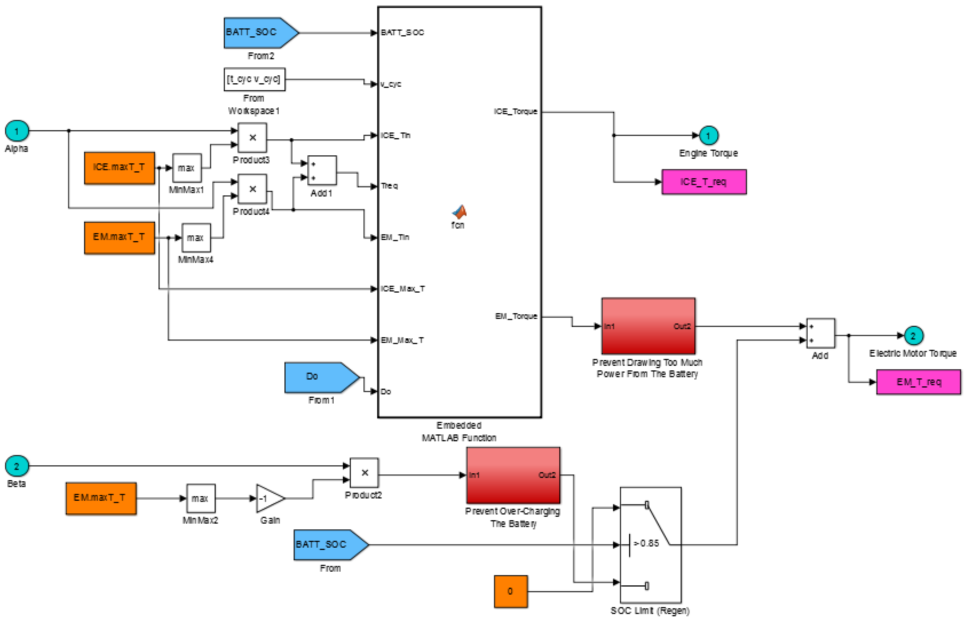


Figure 5

Additionally, the SOC should be kept within a certain range to ensure consistent power delivery and long battery life. Here, the bounds for the SOC are 40% at the minimum and 90% at the maximum. When the state of charge approaches 40%, the control strategy utilizes the internal combustion engine as the primary actuator. As the SOC approaches the upper bound, the power absorbed via regenerative braking is rejected.

Using an electric machine to power the rear wheels allows for the internal combustion engine to only be used at the operating points where it is the most efficient. In this case, the electric machine propels the vehicle from rest to a certain speed. At this point, the internal combustion engine takes over as the primary actuator for vehicle propulsion. This strategy allows the engine to operate with greater efficiency which corresponds to less fuel consumption over a given driving cycle.

Modeling of Vehicle Dynamics

The vehicle dynamics eqations are also modeled. The losses aero dynamic drag, rolling resistance, grade resistance are sustracted from the tractive force. The total force is now equal to mass times accleration. The total force is idvided to get the acceleration, integrating it once gives the velocity, one more integration of it gives the position.

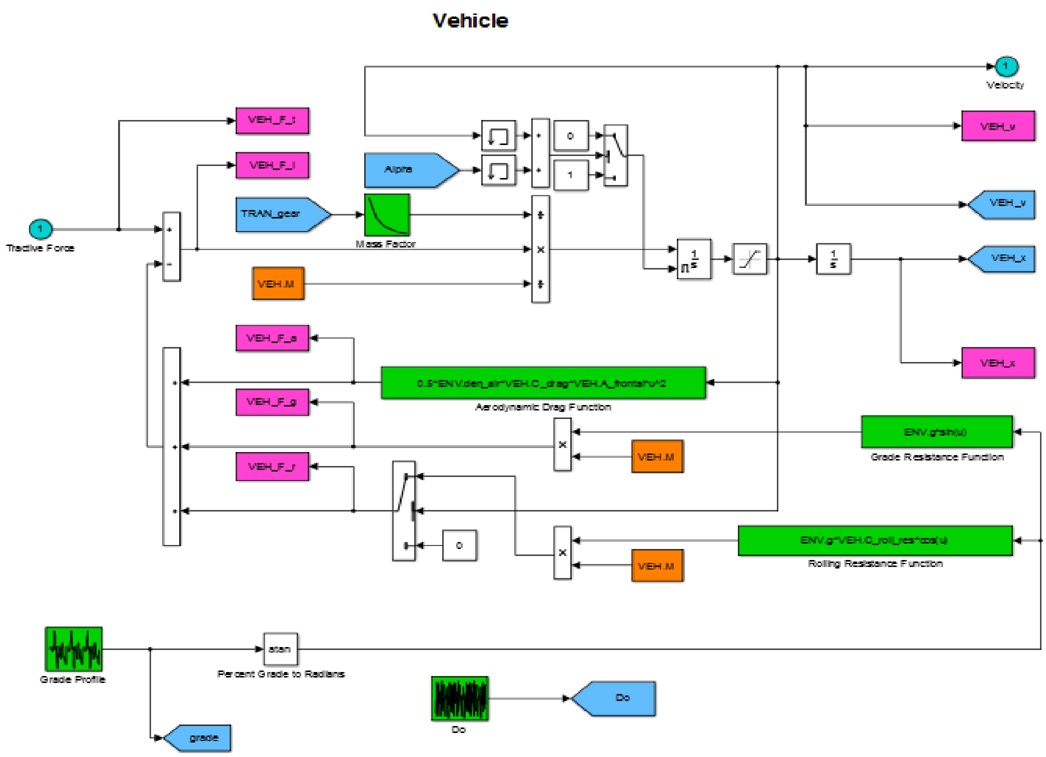


Figure 6

Fuel Economy calculation

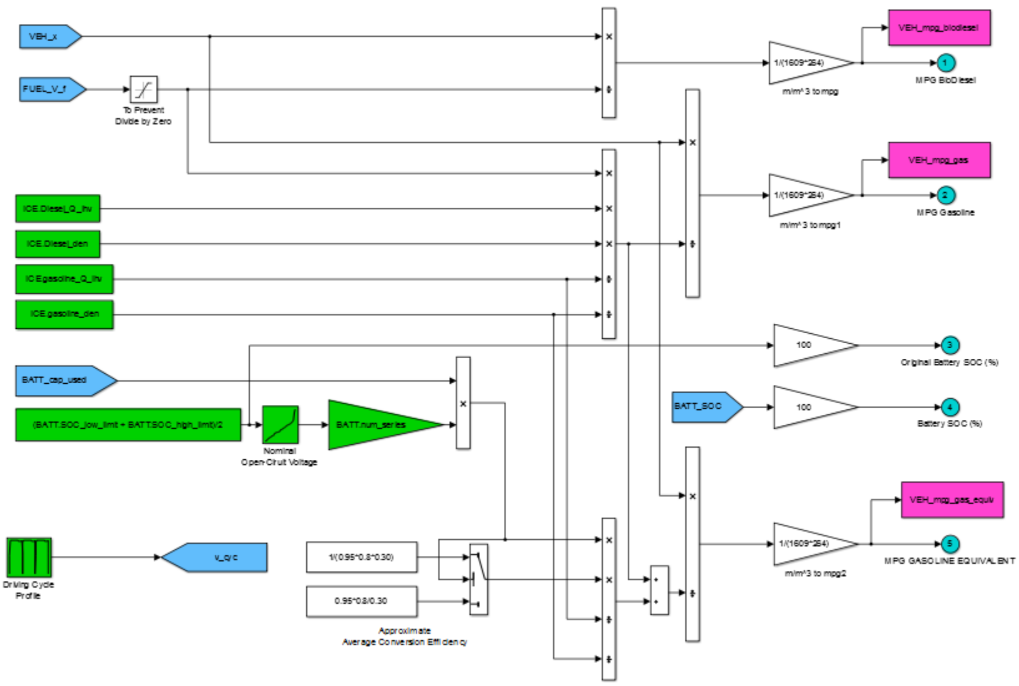


Figure 7

The Testing Route in San Francisco

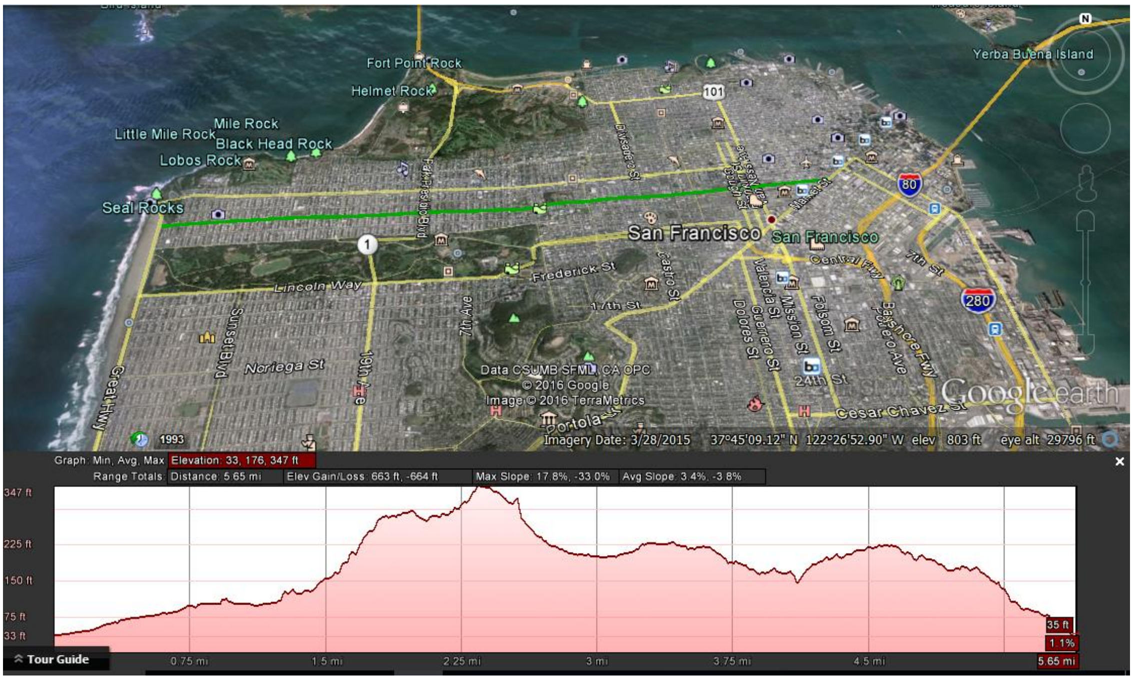


Figure 8 Some awesome caption here? Same for other figs.

The road from San Francisco with the above terrain is chosen, and the data of the elevation of the road with respect to the location is obtained using [9]. This data serves as the main input to the model based on which the mode of fuel usage is going to be decided. One of the important quantities is the grade which is the ratio of rise and run. As already mentioned in dynamic programming we do the optimisation of small sub divisions and then integrate them to find the optimal solution, grade and percentage grade is calculated in the algorithm for each subdivision.

$$\text{Grade} = \frac{\text{Rise}}{\text{Run}}$$

$$\%\,\text{Grade} = \frac{\text{Rise}}{\text{Run}} \times 100$$

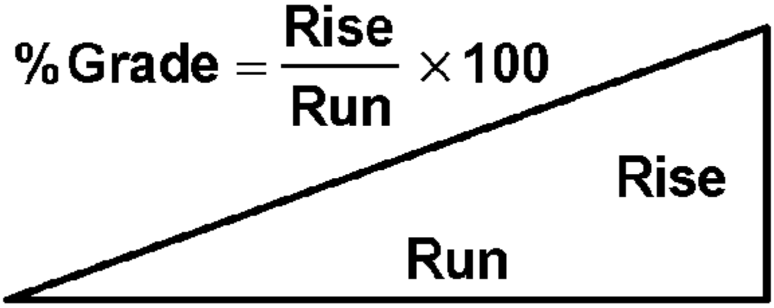


Figure 9

7. NUMERICAL RESULTS

Testing the model for constant cruising drive cycle

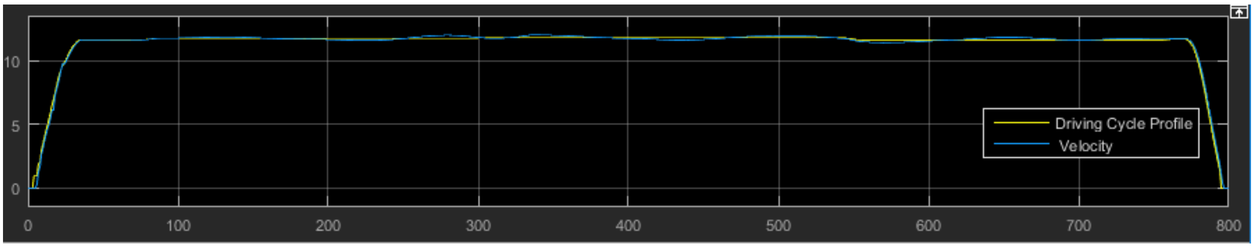


Figure 10

The model was tested for constant cruising speed. The above figure shows the drive cycle for the same. The response in terms of speed matching was found out to be good as from the figure it can be observed that the drive cycle profile and the velocity are mostly on the same level.

CONCLUSION

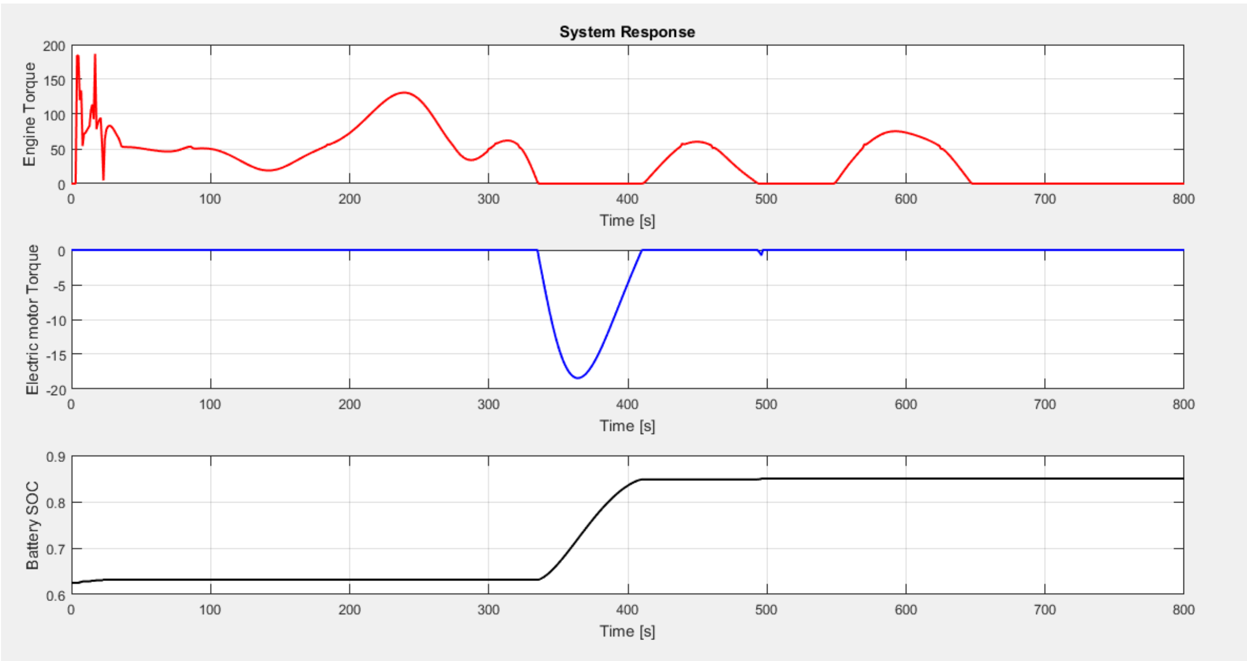


Figure 11

The above figure shows the engine torque, motor torque and the state of charge at any time during the travel. The engine is observed to be mostly used and there was not much energy generated from the regenerative braking. The state of charge initially was at 0.55 and at the end of the cycle it reached 0.88.

Thus the rule based control strategy did not capture enough energy from the regenerative braking and so we are trying to introduce the optimal solution.

8. Dynamic Programming

Bellman's principle

Dynamic programming (DP) as the name itself suggests, solves the problem which varies dynamically (or) with time. The DP is conventionally being used in water dam allocation problems. DP typically uses a cost function, which is the measure of the quantities that we are interested in. It can be said that Dynamic Programming takes a decision at present stage such that the decision results in optimum cost function at now and the optimum cost functions also in the future stages to come M.P. O'Keefe et al [2].

Bellman's principle of optimality

"An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision."

Policy

A policy is the decision taken at each stage in order to optimise the cost function. Optimal or the best policy is the one which optimises the cost function not only in the present stages but also optimises the costs in the future stages that arise due to the policy adopted now.

This explains the characteristic of the DP approach.

Stage

In dynamic programming the optimisation problem is structured in to number of stages of which one stage is solved at a time. These are often formed on the basis of the time. However there are some problems where the DP is implemented but the stages are not time but represents the sequential steps that are to be followed.

The total problem in our project is divided into 800 stages. The variables present in the stages can vary from maximum to minimum in one stage time interval. Thus the IC ENGINE and the motor should be able to run from maximum to minimum or vice versa.

State

State represents the associated variables at each stage. Due to huge computational cost involved in the dynamic programming approach, care should be taken that number of state variables should be as small as possible. Also while doing so it is most import that our state variables represents the system accurately and that the state variables give us enough information to make the future and present decisions. The state variable in this project is the state of charge of battery (SOC).

Cost function

Cost function in dynamic programming refers to quantified objective function. This is calculated in all the stages of the DP. Finally the DP solves for the optimal cost function.

Cost function can be minimisation or maximization depending on the problem. The cost function in the project is Minimization of the flow rate of fuel to the IC Engine.

Cost = cost in the present stage + costs incurred in the future stages.

Infeasibility conditions:

The DP Problem generally involves in number of stages and states. So the amount of computations involved is very high and it increases the cost of computing. Any computations that does not improve cost function is to be avoided by giving conditions called infeasibility conditions. These make sure that the computations does not add any value to our study are avoided. There are infeasibility conditions given to IC Engine, Motor and battery in this project which will be discussed in detailed in the later sections of the report.

While all the constraints and the objective function is the same as in rule based control approach, the objective function is now formulated according to the dynamic programming.

9. Objective function

Our objective is to minimize the fuel consumption which can be formulated as follows

$$\text{Min}\sum_{k=0}^{N-1} \Delta M_f(U_k, K)$$

U is the torque split of the engine.

K is the Kth stage

X is the state of charge

N is the number of stages in the DP problem

Constraints

Subjected to

$$\mathbf{X_{k+1} = f_k(x_k, u_k) + x_k}$$

Initial state of charge $X_0 = 0.65$

Final state of charge $X_N = 0.65$

Lower and upper limit of SOC $0.4 < X_k < 0.9$

No of stages $N = \frac{800}{Ts} + 1$

10. Vehicle specifications

The following are the vehicle specifications which would be constant through the work. These are the essential quantities on which the dynamics of motion would depend upon.

- Mass of the vehicle=2000kg
- Wheel radius =0.3305 mt
- Rolling friction between the wheel and road =176.6 Newton
- Aerodynamic coefficient =0.0416 N-SEC^2/mt^2

- Engine inertia = 0.14 m
- Motor inertia = 0.03 m
- Idling speed = 73.30382858376183 rad/s
- The heating value of gasoline is considered =42500000 joules/kg

INPUTS

The following are the list of inputs used in Matlab program. The major decisions of the program depends on the nature and the quantity of the inputs.

Table 3

Speed	Inp.W{1}
Acceleration	Inp.W{2}
Gear ratio	Inp.W{3}
Grade profile of the path	Inp.W{4}

Angular speed of the wheel **= speed/0.3305 rad.sec**

Angular acceleration of the wheel **= acceleration /0.3305**

The equation giving the torque on the wheel

Torque on the wheel = (torque due to grade+ internal torque+ aerodynamic drag +friction)

Torque due to friction = 9.81*2000*0.009*cosine(angle)

Torque due to grade= 9.81*2000*sine(angle)

Torque due to aerodynamic drag =0.416*speed^2

Torque due to engine/motor = 2000*acceleration*0.3305

Where,

Angle = arctan(grade).

Gear box ratio

The efficiency of the gear box that is considered in the project is 0.92. The gear ratio used in the project. The following is the gear box ratio that is used in the project

Gear ratio = [11.4900 6.5649 4.3101 3.1993 2.3794 1.9002]

Gear box efficiency =0.92

Equation for crankshaft speed (rad/sec) and acceleration (rad/sec^2)
Crankshaft speed = gear ratio* velocity, while gear ratio is >0

Crank shaft acceleration = gear ratio*wheel acceleration.

Crankshaft torque in Nm
Crank shaft torque = torque/gear ratio/efficiency, if wheel torque and gear ratio is positive
= torque/gear ratio *(efficiency), if gear ratio and torque is negative

Torque Split Equations

11. Vehicle subsystems
11.1. IC Engine

Engine torque

The following are the torque values at 8 discretized Levels. These torque values are further used in the interpolation functions in the rest of the program
IC Engine Torque list = [22.24 22.24 22.47 23.37 24.65 27.27 29.85 29.77]

The discretized values of the IC engine angular velocity are as shown below. These values are used in the interp1 function in the matlab.
IC ENGINE Angular velocity
IC Engine speed list = [104.6255 188.5479 272.3342 356.3090 440.1790 502.9166 544.9407 576.5032]

Engine drag torque (Nm) and electric motor drag

The engine drag torque is calculated by interpolation .The maximum of angular velocity of crankshaft and 1st listed angular velocity is selected and then the value is compared with the last listed angular velocity. Now the minimum of this is selected. This is to ensure that the angular velocity falls between the minimum possible and maximum possible values. Now this angular velocity is interpolated by using listed angular velocity and listed torque, the corresponding torque is found out.
Electric motor drag is found by the following equation

IC engine drag torque = (crank shaft acceleration*inertia) +(interpolated IC engine torque)
Electric motor drag = crank shaft acceleration *inertia of motor

Total torque required
The total torque required is split between the IC engine and the Electric motor. The control signal inp.U{1} gives us the torque split ratio between the motor and the engine .If the control signal is 1 then the entire power is provided by the engine ,if it is 0 then the total power is provided by the IC engine.
So, total torque is the sum of torque from Engine (if torque split is not 1), torque from the electric motor and the amplified torque calculated from the crank shaft torque**.**
The following equation explains the same.

Total torque required = Engine torque + Motor torque + crank shaft torque

Torque provided by the engine
Torque provided by the engine is the percentage torque split of the engine multiplied by the total torque required. In some situations the torque provided by the ICE engine can also be used for regeneration. So, in such situations even though the torque required is neative IC engine supplies power to regenerate. Although we are trying to avoid the fuel usage, the eqaution pertainin to the such situation is written below.
IC engine torque

- = (1- control signal)*(total torque required), if angular velocity >0 & torque required >0
- = (1-control signal)*(total torque required),if angular velocity >0 & torque required<0

Torque provided by the electric motor
The torque provided by the electric motor is calculate by multiplying the total torque required and the torque split percentage of the motor. Torque split percentage of the electric motor is same as that of the control signal inp.U{1}

- Electric motor torque = torque split of motor percentage* total torque required

Infeasibility condition

1. When the vehicle is below idling speed and torque produced by the engine is used for charging the electric motor
2. The electric motor is producing torque when the actual torque requirement is negative.

Mathematical representation for the above infeasibility condition

1. Torque of the engine>0 and vehicle speed < 73.30382
2. Torque required <0 and electric motor torque >0

Engine and torque equations

The efficiencies and angular velocity listed values are used to interpolate the efficiency at any given angular velocity.

Internal efficiency of engine at all the 8 levels
IC engine listed efficiencies = [0.413 0.410 0.436 0.435 0.436 0.435 0.430 0.413]

Maximum Engine torque List
[196.0211 267.5240 275.0445 280.3563 281.2652 271.8946 253.9005 234.2969]

Engine efficiency at any level is interpolated by using the above efficiency and angular velocity, interp1 function is used in matlab for this purpose. Interpolation equation is as shown below

Matlab equation
ICE_eff = interp1(ICE_w_list,ICE_eff_list,VEH_w,'linear*','extrap')*0.6

Flow rate of fuel
The amount of fuel consumption is calculated as below

$$mass\ flow\ rate = engine\ torque * angular\frac{velocity}{efficiency_{ICE}}/Calorific\ value$$

Engine maximum torque at any level is interpolated by using the above maximum torque and angular velocity, interp1 function is used in matlab for this purpose
ICE engine torque max
= interpolation of (Crank shaft Speed, listed ICE torque; angular velocity)

Power consumption of ICE
As we know the mass flow rate of fuel now, Power of IC Engine can be calculated by rate of flow of fuel multiplied by calorific value
Power of the engine =mass low rate to the engine*calorific value of the fuel.

Infeasibility condition
The torque of IC engine can never be greater than the maximum torque. This forms the infeasibility condition for the engine
Mathematical relation

1. IC engine torque > maximum possible engine torque.

11.2 Motor and related entities

The angular velocity of the motor is listed in the length of 13 and its torque is listed in the length of 17. These lists will be used for interpolation of efficiency of the motor
EM_w_list = [0 95 190 285 380 475 570 665 760 855 950 1045 1140] (rad/sec)

The torque levels of the motor list (Nm)
EM_T_list = [0 11.25 22.5 33.75 45 56.25 67.5 78.75 90 101.25 112.5 123.75 135 146.25 157.5 168.75 180]

Maximum and minimum torque by the motor

The below is the list of the maximum torque by the motor which is indexed by the speed list

EM_T_max_list = [179.1 179 180.05 180 174.76 174.76 165.13 147.78 147.78 109.68 109.68 84.46 84.46]

It is to be noted that the minimum torque of the motor can be negative and here the minimum torque is considered to be same as the maximum torque in quantity. This rises the following equation

EM_T_min_list = -EM_T_max_list
(or)
EM_T_min_list = - [179.1 179 180.05 180 174.76 174.76 165.13 147.78 147.78 109.68 109.68 84.46 84.46]

Efficiency of the motor

The efficiency of the motor varies with both speed and torque. Thus it gives us a two dimensional lookup format as below. Any value of the efficiency that is to be known is found by two dimensional interpolation of the speed and torque of interest.

.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50	.50
.68	.70	.71	.71	.71	.71	.70	.70	.69	.69	.69	.68	.67	.67	.67	.67	.67
.68	.75	.80	.81	.81	.81	.81	.81	.81	.81	.80	.80	.79	.78	.77	.76	.76
.68	.77	.81	.85	.85	.85	.85	.85	.85	.84	.84	.83	.83	.82	.82	.80	.79
.68	.78	.82	.87	.88	.88	.88	.88	.88	.87	.87	.86	.86	.85	.84	.83	.83
.68	.78	.82	.88	.88	.89	.89	.89	.88	.88	.87	.85	.85	.84	.84	.84	.83
.69	.78	.83	.87	.88	.89	.89	.88	.87	.85	.85	.84	.84	.84	.84	.84	.83
.69	.73	.82	.86	.87	.88	.87	.86	.85	.84	.84	.84	.84	.84	.84	.84	.83
.69	.71	.80	.83	.85	.86	.85	.85	.84	.84	.84	.84	.84	.84	.83	.83	.83
.69	.69	.79	.82	.84	.84	.84	.84	.83	.83	.83	.83	.83	.83	.83	.82	.82
.69	.68	.75	.81	.82	.81	.81	.81	.81	.81	.81	.80	.80	.80	.80	.80	.80
.69	.68	.73	.80	.81	.80	.76	.76	.76	.76	.76	.76	.76	.76	.75	.75	.75
.69	.68	.71	.75	.75	.75	.75	.75	.75	.75	.75	.75	.74	.74	.74	.74	.74

Index/look up table for the efficiency

Table 4

The efficiency of the electric motor is interpolated by using the above lists. As it is a function of two terms interp2 function is used in matlab.
The interpolation uses torque list, angular speed list, efficiency map which we know.
Efficiency is found out for the present torque and angular velocity from the efficiency map listed across torque and angular velocity values.

Infeasibility: electric motor:

The infeasibility conditions of the electric motor are formed by the union of two types of scenarios. The first being when the efficiency corresponding to the torque that is lesser than the minimum possible torque and the second is the efficiency corresponding to the torque that is greater than the maximum possible torque.

Electric motor power

As already known, the power is reduced by the factor of efficiency when it is produced and the power during regenerative braking is lesser than the change in kinetic energy by the factor of efficiency.

The equation foe that is as follows

- Electric motor power = motor torque*speed*efficiency if torque<0
  = motor torque*speed/efficiency if torque >=0

11.3 Battery and variables

State of charge list for the battery
The listed state of charge values for the battery are the key values used in interpolation for calculation of the internal resistance of the battery. The internal resistance of the battery varies depending on the state of charge and also depends on its charging or discharging mode. So the interpolation is done using the listed SOC and listed internal resistance.

Resistance of the battery
Battery has different resistance values during charging and discharging. However both the values of the resistances are indexed by the state of charge condition.

Voltage
The voltage of the open circuit of the battery is also indexed by the state of charge. Thus we also need a list of voltage of open circuits to get the open circuit voltage at all the state of charge values.

The following are all the listed values for the battery.

- State of charge list = [0 0.2 0.4 0.6 0.8 1]

- Battery internal resistance (discharging) = [3.3169 0.7610 0.7137 0.7063 0.6921 0.6759]

- Battery internal resistance(charging) = [0.35 0.50 0.85 1.00 2.00 5.00]

- Open circuit voltage of battery= [329.1200 332.8600 336.5560 339.7680 343.4640 347.8200]

Internal resistance of battery calculation

The internal resistance of the battery is calculated by the interpolation of state of charge list and the discharge/charge lists. When the electric motor is producing power then the battery discharge values of resistance are used and when the motor is storing the power the battery charging resistance values are used.

Internal resistance of battery

- = interpolation of SOC and discharge resistance listed values at present SOC (if motor torque>0)
- = interpolation of SOC and charge resistance listed values at present SOC (if motor torque<0)

Battery limitations

The total capacity of the battery is 23.4 Ampere-hour. In addition to this there is also limitation on the maximum current during and this maximum current is different in both charging and discharging modes.
Total battery capacity =23.4 Ah

- Maximum discharging current of battery = 120A
- Maximum charging current of the battery = 72 A

So the logical equations relating the maximum battery current and the charging and discharging modes will be written as

- Maximum battery current = 120 A, if Power of electric motor is positive
- Maximum battery current = 72A , if Power of electric motor is negative.

Battery voltage calculation

The voltage across the battery for the present state of charge (SOC) is found out by interpolation between the State of charge list and open circuit voltage list.

Battery power consumption

Power consumption of the battery is known from the voltage across it and the current that flows across that voltage.

- Power =voltage*current.

11.4 State of charge

Sate of charge of the battery needs to be updated continuously as we run through the program. The new state of charge is found out by the addition of charge that has been added or removed from the previous state of charge. The previous SOC is over written with the new SOC. The equation for charge addition used is given below

- Charge addition = -(battery current)/ (3600*23.4)

The equation takes care of charging or discharging as the symbol of current varies from charging and discharging.

So the new state of charge SOC = $SOC_{present}$ + {(-battery current)/(23.4*3600)}

Infeasibility conditions for battery

Power from battery, P = voltage across open circuit*Current

But open circuit voltage is less than the original voltage due to internal resistance. So there is a loss in voltage. This loss is equal to current*internal resistance. If ‘R’ is the internal resistance then the mathematical equations are as follows

- Power, P =Voltage(open circuit)*I

  But Voltage (open circuit) = V-IR. So power P = (V-IR)*I. this gives rise to the quadratic equation

  $$P = VI - I^2R$$

  Value for the current can be found by solving the above quadratic equation.

There cannot be a situation where the battery current is greater than the maximum possible current in the battery.
Also the current in the battery cannot be imaginary, making its determinant non negative.
Thus from the equation of current and for its roots to be positive is its second infeasibility condition

1. Magnitude(battery current)>Maximum battery current possible
2. $(\text{Voltage})^2 > 4*(\text{power of motor})*(\text{internal resistance})$

In order to avoid all the imaginary values of state of charge, battery current and power that may creep in due to the calculations, we make all the above entities while updating to the real values.

- New SOC
  = real part of state of charge (SOC)
  = (conjugate (SOC) + SOC)/2

- Battery power
  = real part of Battery power
  = (conjugate (battery power) +Battery power)/2

- Battery current
  = real part of (battery current)
  = (conjugate (battery current) + (battery current))/2

12. COST FUNCTION

The main aim of our project was to reduce the fuel consumption .So the cost function is directly associated with the total fuel consumption.
The cost matrix is constructed for the total travel and it is to be minimized. The cost function measures the amount of fuel being consumed at that instance. For this power at that instance is used along with the calorific value of the fuel to find the amount of mass flow rate of the fuel in to the IC Engine.

Cost = power of engine/42500000
Infeasibility for the cost function
Infeasibility of the cost function is the union of the infeasibility of the electric motor, infeasibility of the IC Engine, and the infeasibility of the battery.

Output

The following are the outputs stored under the structure OUT. The torque of the engine, internal resistance in the battery, torque of the electric motor, crank shaft angular speed, battery power , battery current and maximum torque of the ICE are the essential outputs to be able to go to the next stage

Table 5

OUT.ICE_T	IC Engine Torque
OUT.EM_T|	Electric motor torque
OUT.ICE_T_r	Reversed ICE engine torque
OUT.ICE_T_MAX	Maximum torque by the IC engine
OUT.VEH_w	Angular velocity of the crank shaft
OUT.BATT_i	Battery current
OUT.BATT_P	Battery power

13. Results

The following series of plots of the overall outcome can be observed to conclude upon that the dynamic programming technique is a better optimization technique than the rule based control technique.

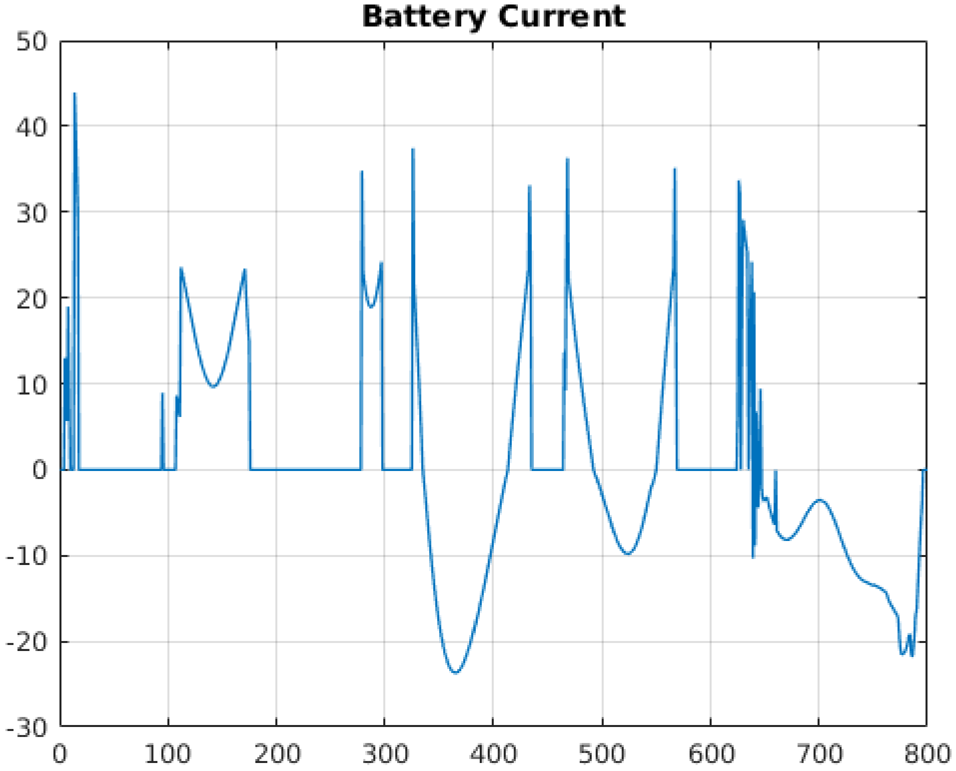


Figure 12
Plot showing the battery current levels across 800 stages

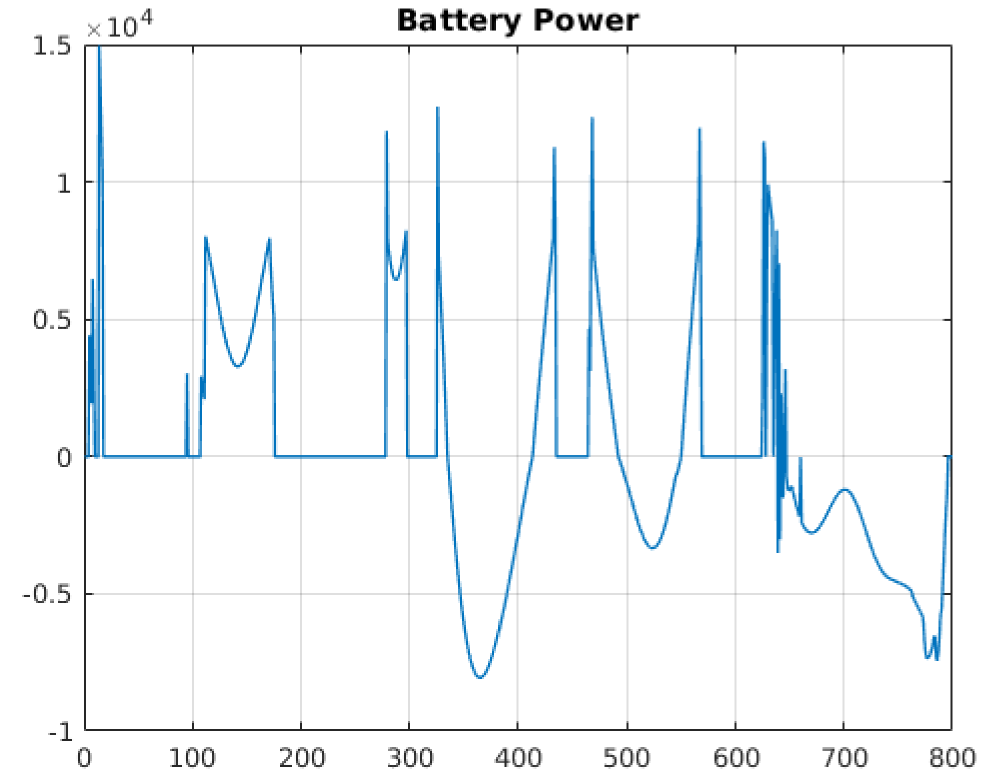


Figure 13: Plot showing the battery power levels across 800 stages

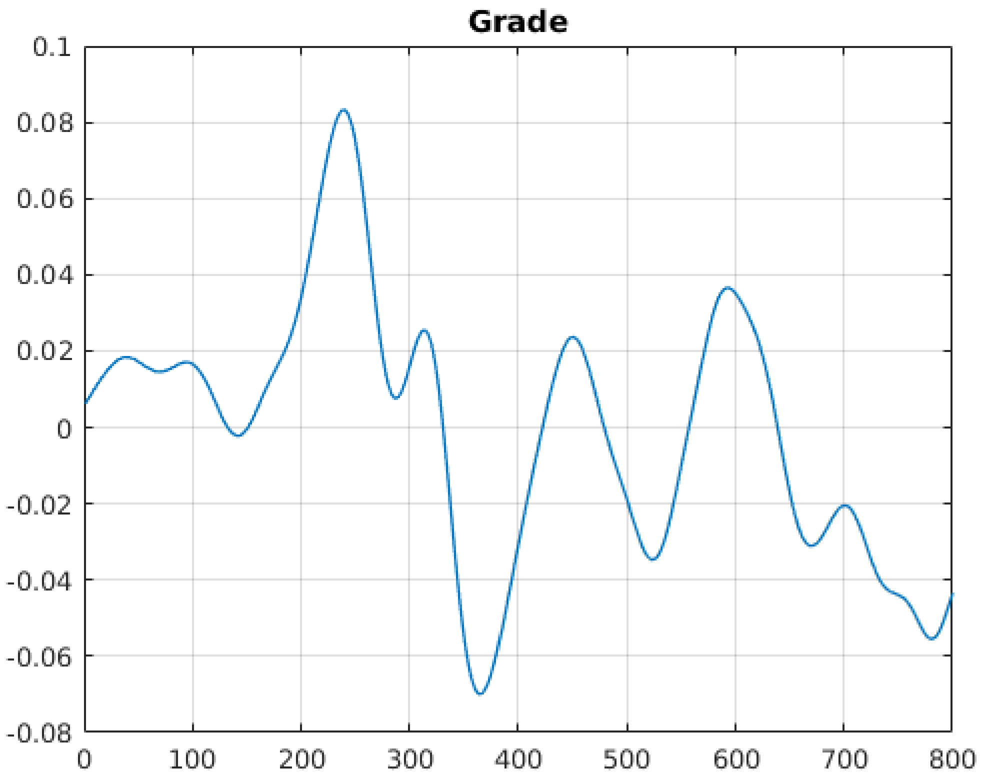


Figure 14
The above plot shows the grade profile of the path to be followed.

The state of charge graph actually gives us the idea of the battery charge. The initial and final condition of state of charge here is maintained at 0.65. The graph can also give the amount of time in which the electric motor is used to drive the vehicle and the time for which it is being regenerated. From the graph one can see that in the total 800 stages, the vehicle used electrical energy to drive in approximately 475 stages and the power was regenerated across approximately 170 stages.

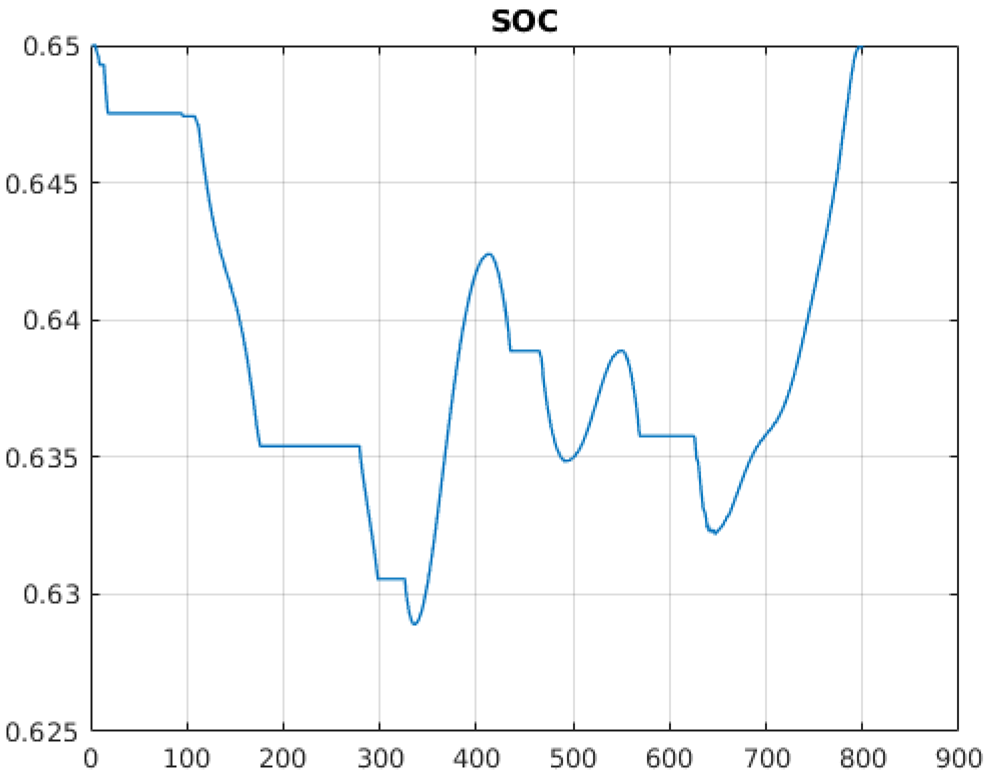


Figure 15
State of charge of the battery along the 800 stages is plotted in the above figure

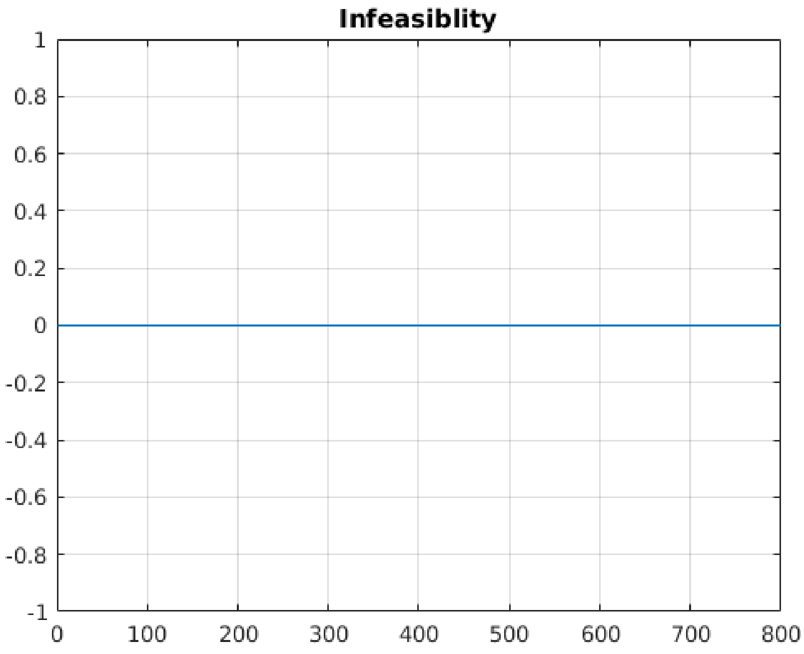


Figure 16
Plot showing the infeasibility condition through all the stages
From this plot it can be made sure that an undesirable combination has never occurred.

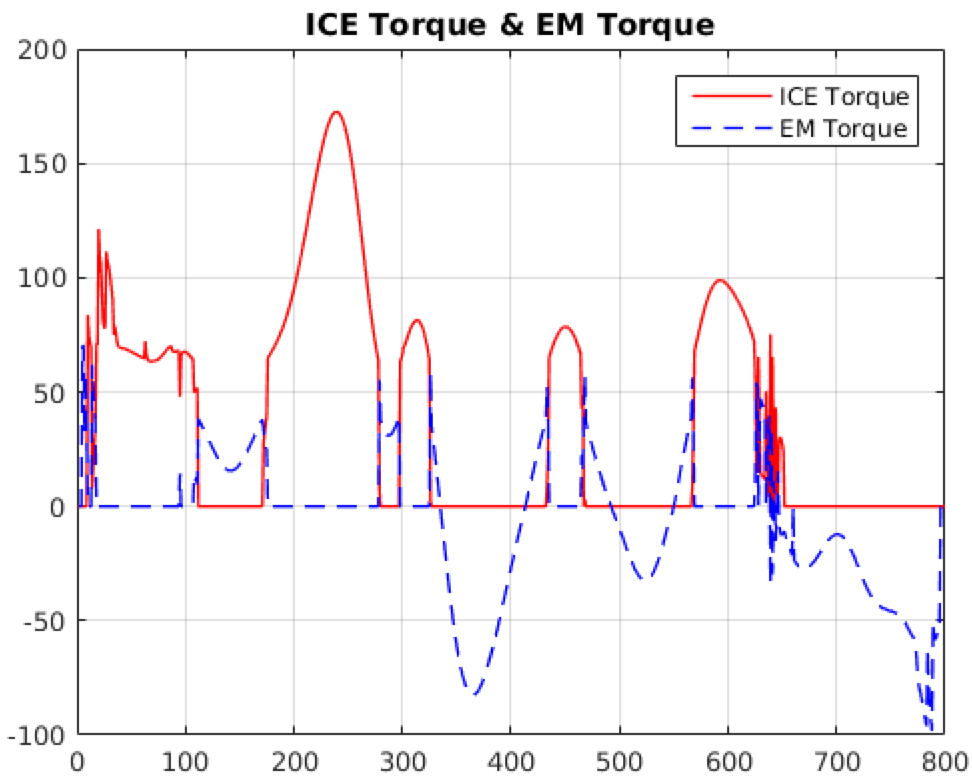


Figure 17
Figure describing the torque split ratio between the electric motor and the IC Engine

The effective dynamic program takes care of the torque split ratio. From the figure it can be observed that at any point of time if the electric motor is being regenerated the IC Engine is supplying no torque, i.e, the undesirable situation of Engine producing the torque and the motor storing the charge is not encountered. Also depending on the state of charge engine gets significant contribution from the motor in providing the torque.

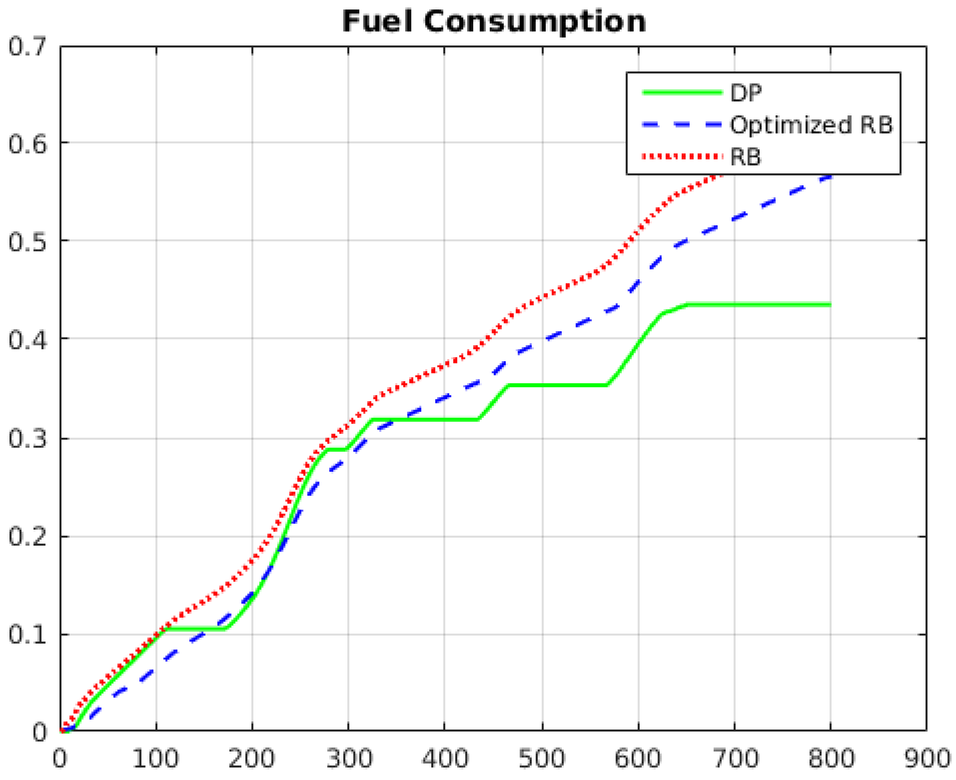


Figure 18
Plot showing the fuel consumption of rule based and DP at all the stages

The fuel consumption which is also our concern or the cost function is plotted across 800 stages. DP has the lowest fuel consumption as can be seen from the graph.

Work Split in the Team

Literature survey and problem formulation	Dheeraj and Mohammad
Simulink modeling for baseline validation	Mohammad
Data acquisition and processing for DP	Dheeraj
DP problem set up	Mohammad
Vehicle dynamics modeling for DP	Dheeraj and Mohammad
Validation of DP results	Dheeraj and Mohammad

⊗ This work can be published w/ some polish and hardware validation
Good job! It would also be interesting if you can collaborate with the traffic jam team.
to consider both grade and traffic info in optimal control. Talk to Ruijin Cang if interested. He's my student.

14. References

[1] Magnus Neuman, Henrik Sandberg and Bo Wahlberg "Rule-Based Control of Series HEVs Derived from Deterministic Dynamic Programming".

[2] M.P. O'Keefe and T. Markel. "Dynamic Programming Applied to Investigate Energy Management Strategies for a Plug-in HEV". Presented at the 22nd International Battery, Hybrid and Fuel Cell Electric Vehicle Symposium and Exhibition (EVS-22) Yokohama, Japan October 23–28, 2006.

[3]Kamil Çagˇatay Bayindir, Mehmet Ali Gözüküçük, Ahmet Teke .A comprehensive overview of hybrid electric vehicle: Powertrain configurations, powertrain control techniques and electronic control units.
http://www.sciencedirect.com/science/article/pii/S0196890410004309.

[4]Michael Arnett .Modelling and Simulation of a Hybrid Electric Vehicle for the Challenge X Competition **https://kb.osu.edu/dspace/bitstream/handle/1811/326/Arnett_Thesis.pdf**

[5]Leon Yuliang Zhou .Hybrid Electric Vehicle Powertrain and Control System Modeling, Analysis and Design Optimization.

https://dspace.library.uvic.ca:8443/bitstream/handle/1828/3714/Zhou_Leon_PhD_2011.pdf?sequence=5&isAllowed=y

[6] Optimization of power management in an hybrid electric vehicle using dynamic programming. http://www.sciencedirect.com/science/article/pii/S0378475406001807

[7] Application of genetic algorithm for optimization of control strategy in parallel hybrid electric vehicles. http://www.sciencedirect.com/science/article/pii/S0016003206000329

[8] Design optimization and optimal control for hybrid vehicles. Optim Eng https://www.researchgate.net/publication/225593667_Design_optimization_and_optimal_control_for_hybrid_vehicles_Optim_Eng

[9] The following references were used to get the elevation data . google earth and http://www.geocontext.org/publ/2010/04/profiler/en/

[10] http://www.idsc.ethz.ch/

[11] Rui Wang , Srdjan M. Lukic "Dynamic Programming Technique in Hybrid Electric Vehicle Optimization" **http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=6183284**